Analytic Functors

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The finitary analytic functors introduced by Andre Joyal are defined as those endofunctors of $\textbf{Set}$ which are left Kan extensions of functors from $\mathbf{B}$, the category of all finite sets and isomorphisms, into $\textbf{Set}$. They can be characterized as precisely the finitary set functors weakly preserving countable wide pullbacks. And they are described as all coproducts of finitary pseudo-representables. Here we call, for every object $A$ of a category $\mathbf{K}$, the quotient of the hom-functor of $A$ modulo a group on automorphisms of $A$ a pseudorepresentable.

We generalize analytic functors to include functors between locally presentable categories $\mathbf{K}$ and $\mathbf{L}$: Let $\mathbf{B}_\lambda$ be the category of all $\lambda$-presentable objects of $\mathbf{K}$ and all isomorphisms. The $\lambda$-ary analytic functors from $\mathbf{K}$ to $\textbf{Set}$ are precisely the left Kan extensions of functors from $\mathbf{B}_\lambda$ to $\textbf{Set}$. They are characterized as precisely the $\lambda$-accessible functors weakly preserving wide pullbacks. And they can be described as the coproducts of pseudo-representables. Finally, a functor from $\mathbf{K}$ to $\mathbf{L}$ is called analytic iff its composite with every hom-functor of $\mathbf{L}$ is analytic. These are precisely the accessible functors weakly preserving wide pullbacks.

Example: analytic endofunctors of the category of graphs are precisely the liftings of analytic endofunctors of $\textbf{Set} \times \textbf{Set}$ via the canonical forgetful functor.

We present a comparison to other generalizations of analytic functors due to M. Fiore [1] and M. Abbott et al. [2].

References:

[1] M. Fiore: Exact characterisation of generalized analytic functors between groupoids, a preprint
