SPREADING OF WAVE PACKETS IN ONE DIMENSIONAL DISORDERED CHAINS: DIFFERENT DYNAMICAL REGIMES

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Abstract

We present numerical results for the spatiotemporal evolution of a wave packet in quartic Klein-Gordon (KG) and disordered nonlinear Schrödinger (DNLS) chains, having equivalent linear parts. In the absence of nonlinearity all eigenstates are spatially localized with an upper bound on the localization length (Anderson localization). In the presence of nonlinearity we find three different dynamical behaviours depending on the relation of the nonlinear frequency shift \( \delta \) (which is proportional to the system's nonlinearity) with the average spacing \( \Delta \) of eigenfrequencies and the spectrum width \( \Lambda (\Delta \chi A) \) of the linear system. The dynamics for small nonlinearities \( \delta < \Delta \) is characterized by localization as a transient, with subsequent subdiffusion (regime I). For intermediate values of the nonlinearity, such that \( \Delta \chi A < \Delta \) the wave packets exhibit immediate subdiffusion (regime II). In this case, the second moment \( m_2 \) and the participation number \( P \) increase in time (following the power laws \( m_2 \sim \tau, \ P \sim \tau^2 \)). We find \( \tau 1/3 \). Finally, for even higher nonlinearities \( \delta > \Delta \) a large part of the wave packet is selftrapped, while the rest subdiffuses (regime III). In this case \( P \) remains practically constant, while \( m_2 \sim \tau \).

Models and computational methods

We study [1] two models of one-dimensional lattices:

**The quartic Klein – Gordon (KG) model**

\[
H = \sum_{l} \left( \frac{p_{l+1}^2}{2} + \frac{e_{l+1}^2}{2} w_{l+1}^2 + \frac{1}{4} u_{l+1}^4 + \frac{1}{2W} (u_{l+1} - u_l)^2 \right)
\]

where \( u_l \) and \( p_l \) are respectively the generalized coordinates and momenta, \( l \) is the lattice site index, \( W \) is the disorder strength, \( \delta \) the total energy and typically \( \hbar = 1 \).

\( e_{l+1} \) are chosen uniformly from \( \left[ \frac{1}{2} , \frac{3}{2} \right] \).

The disordered discrete nonlinear Schrödinger (DNLS) equation (see also poster 14)

\[
H = \sum_{l} \left[ \frac{p_{l+1}^2}{2} + \frac{e_{l+1}^2}{2} w_{l+1}^2 \right] - \frac{1}{2W} (u_{l+1} - u_l)^2
\]

with complex variables \( \psi_l \). The random on-site energies \( e_l \) are chosen uniformly from \( \left[ \frac{1}{2} , \frac{3}{2} \right] \).

Linear case of the KG model (neglecting the term \( u_l^4 \))

**Ansatz:** \( u_l = \psi_l \exp(\imath \alpha) \), \( \psi_l \in \mathbb{C} \)

**Eigenvalue problem:** \( \lambda \psi_l = e_{l+1} - (e_{l+1} + A) \psi_l \), with \( \lambda = \text{Re}(\omega) - W - 2\), \( e_l = \text{Re}(\omega) - 1\).

Unitary eigenvectors (normal modes - NMs) \( \psi_l \) are ordered according to their center-of-norm coordinate: \( X_l = \sum_{l} |\psi_l|^2 \).

All eigenstates are localized (Anderson localization) having a localization length which is bounded from above.

- **Scales**
  - Linear regime: A linear wave packet that spreads downwards with increasing nonlinearity.
  - Subdiffusion: Assuming that the spreading is due to heating of the cold exterior, induced by the chaoticity of the wave packet, we theoretically predict \( \tau 1/3 \).
  - Intermediate values of nonlinearity: Resonance overlap may happen immediately. Immediate subdiffusion [3].

- **Nonlocal excitations**
  - Frequency shift exceeds the spectrum width. Some frequencies of NMs are tuned out of resonances with the NM spectrum, leading to selftrapping, while a small part of the wave packet subdiffuses [4].

**Different spreading regimes**

- **Regime I:** Small values of nonlinearity, \( \delta < \Delta \). Resonance overlap may happen immediately. Immediate subdiffusion [3].
- **Regime II:** Intermediate regime of nonlinearity, \( \Delta \chi A < \Delta \). The wave packets exhibit immediate subdiffusion (regime II)
- **Regime III:** For large \( \delta > \Delta \) a large part of the wave packet is selftrapped, while the rest subdiffuses (regime III).

**Distribution characterization**

We consider normalized energy distributions in normal mode (NM) space \( z_l = \sum_{l} |\psi_l|^2 \) with \( \zeta = \sqrt{\sum_{l} |\psi_l|^2} \), where \( \zeta \) is the amplitude of the lth NM.

- **Second moment:** \( m_2 = \sum_{l} \langle \psi_l \rangle \left( \frac{1}{2} \sum_{l} |\psi_l|^2 \right) \zeta \), with \( \zeta = \sum_{l} |\psi_l|^2 \) quantifies the wave packet's degree of spreading.
- **Participation number:** \( P = \frac{1}{\sum_{l} |\psi_l|^2} \) measures the number of stronger excited modes in \( z_l \).
- **Compactness index:** \( \zeta = \frac{P}{m_2} \) measures the sparseness of wave packets.

The KG chain was integrated with the help of a symplectic integrator of order \( \mathcal{O}(\tau^2) \) with respect to the integration time step \( \tau \), namely the SABA3 integrator with corrector (SABA3_C) [2].


REFERENCES