

BERNHARD BANASCHEWSKI DOCTOR HONORIS CAUSA

A PERSONAL LAUDATIO

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The University of Cape Town is awarding the honorary degree of *Doctor of Science honoris causa* to Professor emeritus doctor rerum naturalium Bernhard Banaschewski of McMaster University in Hamilton, Ontario. The selection process is demanding, but the laureate is indeed such a distinguished scholar that one cannot but wholeheartedly applaud the award to this polished academic citizen recognized all over the world. Any institution citing BERNHARD BANASCHEWSKI for his scholarly work, for his contributions to mathematics, and for his influence on generations of its students will bring honour onto itself.

Some initial personal notes

BERNHARD BANASCHEWSKI and I met in the summer of 1954 at the University of Hamburg, where I studied under COLLATZ, HASSE, PASCUAL JORDAN, SPERNER, WITT. In particular, I took a seminar with the legendary ERNST WITT which was organized and managed by his assistant: none other than BERNHARD BANASCHEWSKI; the topics included a study of ANDRÉ WEIL's book "L'intégration dans les groupes topologiques et ses applications"; that exposure gave my mathematical life its direction. Even today I recall a colloquium lecture delivered that summer by BANASCHEWSKI to the Hamburg mathematicians on ordered topological groups; he compared the order completion and the uniform completion; this research was later published in *Mathematische Nachrichten* 16(1957). It was during that summer that at the mathematics department of the University of Hamburg people began to speak about his plans to emigrate to Canada. This plan materialized and thereafter, when BANASCHEWSKI's and my paths crossed again years later, I rarely ever heard him speak German again.

Some historical notes on postwar mathematics in Germany

In order to depict the life of a scholar, like BANASCHEWSKI's, one cannot but see it embedded into its historical environment. BERNHARD BANASCHEWSKI is a member of what one might call the *first generation* of German mathematicians after World War II; born around 1925 they were fed into the Nazi war machine as teenagers, serving as under-age anti-aircraft gunners and as half-trained infantry in the last phases of a lost war. When BERNHARD BANASCHEWSKI began to visit Tulane University in the early sixties — which he continued doing for many years to come — the algebraist PAUL CONRAD, who was a tail gunner in one of the flying fortresses over Germany in World War 2, and Bernhard determined that they had been shooting at each other twenty years earlier. Those young Germans who returned from the war entered the universities (or what was left of them) in 1945 and subsequent years; highly motivated, having one-track minds in order to get over what they experienced, they received their degrees around 1950, learning from teachers

surviving from the heydays of German mathematics in the twenties and the early thirties like the ones at Hamburg I mentioned, or, coming to my mind, BEHNKE, HELMUTH KNESER, KAMKE, KNOPP, KÖTHE, OTTHEINRICH KELLER and their likes. That generation graduating in the early fifties filled a void left by the loss of practically one entire generation in the war, and it moved into the professoriate in the fifties and early sixties. The German mathematicians of this first generation after the war joined their peers in other nations to rebuild mathematical research in the second half of the century, some continuing where BOURBAKI had started in France around 1940, and some picking up the threads left dangling from the times before the Nazi onslaught and the war. Some of this first generation German mathematicians became quite well known; BANASCHEWSKI, HEINZ BAUER, DOLD, GRAUERT, HIRZEBRUCH, HUPPERT, JACOBS, MARTIN KNESER, KRICKEBERG, LEOPOLD, LEPTIN, PUPPE, REMMERT, ROQUETTE, SCHAEFER, ZELLER come to my mind as those I perceived as a young mathematician of the *next* generation, a generation trailing these by half a decade or a decade.) Almost all of the first and second generation mathematicians spent some time abroad, mostly in North America, some for a very long time in human terms (like myself, joining Tulane University in New Orleans) and some to stay for good (like BANASCHEWSKI, joining the faculty of McMaster University in Hamilton). Only in the later sixties did the academic employment market in Germany open up as a consequence of the structural reform and the expansion of the German university system.

Meeting Banaschewski in the US

So my acquaintance with BERNHARD BANASCHEWSKI spans a period of forty-six years. The distance between us, mathematically and geographically, is a function with bumps and dents; we share an early interest in general topology and topological algebra, notably in the domain of topological groups. Another area of contact emerged in the form of category theory which is pervasive in BANASCHEWSKI's work and which both he and I approached as an important language and a tool for other things we had in mind rather than as a field of study in its own right; his contributions to the theory of sheaves in the category of Banach spaces [*Quaestiones Mathematicae* 2(1977)] touched me at a time when DAUNS, KEIMEL, MULVEY and I were working on the use of general bundles or sheaves for the purposes of representing rings and algebras; and finally we got rather close again during the period that BERNHARD BANASCHEWSKI on the one hand and GIERZ, KEIMEL, LAWSON, MISLOVE, and DANA S. SCOTT and I on the other worked on continuous lattices. BANASCHEWSKI's papers in the *Houston Journal of Mathematics* 4(1978), the *Canadian Journal of Mathematics* 32 (1980) reflect that interest, and the *Springer Lecture Notes in Mathematics* of 1981 which he jointly edited with RUDOLF-EBERHARD HOFFMANN of the University of Bremen quickly became one of the two comprehensive sources on continuous lattices. The entire activity was of an "interdisciplinary" nature: The logician DANA SCOTT, whose declared goal at the time was to formulate a mathematical foundation of computing and of a denotational semantics of programming languages, had introduced and baptized "Continuous Lattices" around 1970 in this pursuit. At a lattice theory conference in Houston in 1973 BERNHARD BANASCHEWSKI delivered a lecture in which he alerted the audience to SCOTT's lead article which had appeared 1972 in the Proceedings of the Dalhousie category theory conference, edited by F. W. LAWVERE. BANASCHEWSKI's comment soon proved to be crucial for the initiation of the collaboration of DANA SCOTT with algebraists and functional analysts who had come across continuous lattices in quite different guises; I shall return to this

important contribution of BANASCHEWSKI's again.

During the sixties and seventies Bernhard visited New Orleans regularly, at least once a year, often mathematically, sometimes socially. It was a firm agreement that he would come and visit with my wife Isolde and myself at our house. One time he was a visiting professor for a year at Tulane University teaching a course on such matters as free topological groups; if my memory serves me right this was in 1967-68. I regretted that his presence at Tulane coincided with my absence on a sabbatical leave. During the last 18 years I have been in Germany, keeping up regular visits to New Orleans myself, and I was mainly working in Lie groups and semigroups so that the mathematical distance between Bernhard and myself increased again; I did, however, see him once or twice in Germany and again most recently at an international conference on category theory in Bremen.

Banaschewski's style and influence

BANASCHEWSKI had an enormous influence on the development of those fields he worked in: General topology, lattice and order theory, mathematical logic, and category theory. He is a great stylist and aesthete, mathematically and socially; his presentation in writing and as an orator is impeccable. His preferred form in mathematical writing is the succinct and elegant article. Had he been a literary writer he would not have written novels, but short stories, poetry and essays, and had he been a composer, symphonies would not have been his business. Nonetheless he is quite present in literature collected between two hard covers; numerous authors describe and cite his work in their books just as my co-authors and I have done in the "Compendium of Continuous Lattices" of 1980. In a similar vein, BERNHARD BANASCHEWSKI is prominent in the category theory and order theory community through the collections he edited, notably the ones in the Springer series *Lecture Notes in Mathematics*. These have been quoted frequently up to the present time.

In mathematics, it may happen to authors that they pass through periods of several years without an utterance of creative work in public. Not so BANASCHEWSKI! His bibliography boasts a proud list of some 140 publications which must average him about three articles per year, a publication record which is nothing less than outstanding, notably if we consider the seminal characters of many of his papers. The span of 48 years in the history of his publications neatly reflects the growth of the market place of mathematical journals. In the first decade his publications appeared in a comparatively narrow range of journals—mostly in the *Mathematische Nachrichten* (then published in Berlin by the Akademie der Wissenschaften der Deutschen Demokratischen Republik, in due time becoming the leading mathematical journal of that country), *Archiv der Mathematik* (Basel), *Canadian Journal of Mathematics*, *Canadian Mathematical Bulletin*; later his publications spread over a wide variety of international and specialized journals such as *Algebra Universalis*, *Journal of Pure and Applied Algebra*, *Order*, *Mathematical Proceedings of the Cambridge Philosophical Society*, and he published a series of articles in *Quaestiones Mathematicae*, connected with the University of Cape Town, which BERNHARD BANASCHEWSKI visited regularly and for extended periods during his later professional life. I remember him often to have spoken about South Africa with affection and fairness. The spread of BANASCHEWSKI's papers reflects the impressive scope of his mathematical erudition. The presence of his knowledge of so many different subjects in our field has

struck me as being close to encyclopedic. His style of writing is distinguished by certain invariable characteristics. For instance, where most writers will present a "Theorem" or even a "Main Theorem", Bernhard with consistent modesty speaks of a "Proposition"; he admits himself that a referee once stopped him having submitted a paper entitled "A Theorem on *dots*" having no "Theorem" in the text. His readers, knowing his style, have of course learned to look for his major results under the heading of "Propositions".

BANASCHEWSKI'S influence on mathematical research was pervasive due to the substance and elegance of his mathematics, and to the persuasion with which he presented it at conferences. BANASCHEWSKI is one of the outstanding and unforgettable characters to those who have met him, a person of great social radiance and charm, and of a cutting sharpness of intellect delivered with an articulation of crystal clarity.

Banaschewski's mathematical work

It would be difficult, if not impossible, to do justice to the volume of mathematical work which BERNHARD BANASCHEWSKI published in the course of a prolific life as a scholar. A few general observations which I am going to make from my own vantage point may still be in order.

What was rather novel to the preceding generation of mathematicians in the late forties and early fifties was the spirit of BOURBAKI which was generated by two dozen slender books published one after another since about 1940 by a group of French mathematicians under the pseudonym of NICHOLAS BOURBAKI. They attempted to recast mathematics with rigorous consistency, building mathematical structures from scratch in greatest generality possible, and with the intent to derive the amenities of day-to-day mathematics by descending from the peaks of generality to the netherlands of particular situations and special cases at the foot of Mount Olympus. This undertaking required visionaries (which they were), the creation of powerful and effective concepts, and a heretofore unparalleled discipline in remaining on track over such a long haul. This enterprise endured until the seventies, when the initiators' ideas and the energy of those supposed to carry the torch forward expired. But the young mathematicians in the late forties and the fifties were fascinated, and most of them retained the love for conceptual thinking throughout their whole lives. I believe that BERNHARD BANASCHEWSKI is one of them. His style of mathematics is that of conceptual reasoning as opposed to algorithmic or computational proceeding. His avocation is structural mathematics, not problem solving. Within this consistent line, amazingly, he was able to publish, in 1982, with EVELYN NELSON a paper in the *SIAM Journal of Computing*. Indeed it has become an accepted fact among people working in theoretical computer science today that order theory and topology (as well as category theory indeed) provide, abstract as they are, the essential tools in the semantics of programming languages and in the theory of computation.

From the fifties onward, BERNHARD BANASCHEWSKI contributed to general topology, order theory, and algebra, as well as to their applications in mathematical logic. It is consistent with his style that he should soon be attracted to category theory from the view point of applications to topology, algebra, and logic. In the sixties, BERNHARD BANASCHEWSKI was the first to give, in this framework, a comprehensive theory of injectivity and projectivity

in arbitrary categories. Like a leitmotiv, injectivity and projectivity in various algebraic or topological categories recur in his work. BERNHARD BANASCHEWSKI has an exceptionally comprehensive knowledge of existing literature pertaining to the subjects of his research, and the analytic intellect to perceive and recognize fruitful new developments long before people hop onto the bandwagon. So he was among the first to observe, in the domain of topology, algebra and lattice theory combined, that DANA S. SCOTT had taken a remarkable step, when he discovered and first discussed *continuous lattices* around 1972 in a collection dedicated to category theory and edited by LAWVERE. In computer science SCOTT's discovery had been recognized for the breakthrough it was, because in that subculture SCOTT's work on the denotational semantics of programming languages and in the theory of computing was well known. But it took a few years until the mathematical community recognized that there was a rich mine connecting lattice theory, topological algebra, and topology, waiting to be exploited. BANASCHEWSKI was one of the first to prospect, and he remained a leading figure in the mathematical development of continuous lattices. At about the same time when he began to write on continuous lattices, he was able to give a conclusive impulse to the theory of sheaves of Banach spaces; this contribution was published a bit later in his *Quaestiones* paper and was discussed on various conferences and in several proceedings volumes. How he could accomplish these different lines of investigation almost simultaneously remains a mystery to me; other people (myself included) had been thinking about such matters for years without having exactly this right idea.

Among the working topologists and lattice theoreticians, BANASCHEWSKI was one of the first to embrace the theory of locales which, as an aspect of topos theory coming originally out of algebraic geometry, was soon a proper domain of the category theoreticians of the logic oriented colour. Locale theory is a theory of topological spaces and continuous functions dealing with spaces without points. It therefore is certainly one of the most abstract theories mathematics has spawned in the second half of this century. Much of BANASCHEWSKI's later work up to this day culminated in this area. Here his early interests in the juncture of topology, order theory and logic merged to a mature synthesis. Along this line of thinking BANASCHEWSKI dealt in enlightening ways with logical problems involving the Axiom of Choice. The role of this axiom, that most working mathematicians in, say, functional analysis will accept without flinching, is quite subtle from a logical point of view, and BANASCHEWSKI's recurring interest focussing onto this theme has brought many surprising aspects to the light. It has been a puzzler to me that many a basic theorem in topology (such as e.g. Tychonov's Theorem on the product of compact spaces), solidly resting on the Axiom of Choice or one of its near neighbours, has a localic generalisation whose proof does not require the Axiom of Choice. Apparently this axiom is — not evident on the surface! — only needed when one really wants to find "this point" in the product; in the localic phantom version, by its very nature, the presence of points is not an immediate concern. JOHN ISBELL and CHRISTOPHER MULVEY have often, each separately, tried to teach me to think locally; this entails pretending that certain functions are not functions but arrows pointing the other way and in this fashion become (phantom) continuous functions between spaces that have no points. We have really come a long way from BOURBAKI with a tall order like this, and I have never been quite able to become fully comfortable with this concept even though, with some of my own interests twenty years ago, I have come very close. With pleasure I record the fact, that whenever I hear

BERNHARD BANASCHEWSKI lecture on his ideas in locale theory I have the warm feeling of understanding; he does not even use the word "locale" and insists on speaking of frames and frame morphisms, a terminology going back to DOWKER. Of course, all locale theoreticians know what they are doing and lecture competently about their subject, but BERNHARD BANASCHEWSKI, in his way of presenting this material, never leaves the category in which he works, and I, a lover and practitioner of duality theories, always have the confidence that I am listening to a working mathematician who can get his ideas across without forcing me to bend arrows around one hundred and eighty degrees.

BERNHARD BANASCHEWSKI : What a great pedagogue he is — on top of being a brilliant mathematician! I offer my felicitations to him on the occasion of the well deserved honour that the University of Cape Town is bestowing upon him.

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