Accurate Estimation of Risk When Constructing Efficient Portfolios for the Capital Asset Pricing Model

A thesis presented by

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Abstract

In this paper, we investigate the behaviour of the efficient frontier and optimal portfolio of the Troskie-Hossain Capital Asset Pricing Model (TrosHos CAPM) and Sharpe Capital Asset Pricing Model (Sharpe CAPM) when the covariance structure of the residuals is correlated under the Markowitz formulation. By building in the dynamic time series models: AR, GARCH and AR/GARCH we were able to model the autocorrelation and heteroskedasticity of the residuals. The study extends Hossain et al. (2005) who carried out a similar investigation but did not incorporate the CAPM model assumptions on the TrosHos and Sharpe single index models. Our evidence displays that the TrosHos CAPM model gives a more accurate account of the risk in a portfolio when the covariance structure of the residuals are correlated. The Sharpe CAPM model tends to either underestimate or overestimate the risk inherent in a portfolio when the covariance structure of the residuals is correlated.

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Declaration

I declare that this thesis is my own work and any information received from other sources has been cited. It is being submitted in partial fulfillment of the degree of Master of Science in Mathematics of Finance to the University of Cape Town, South Africa. It has not been submitted before for any degree or examination to any other university.

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Date: 4 November 2010

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1 Introduction

1.1 Background

In his seminal paper entitled, "Portfolio Selection" Markowitz laid out the foundation for Modern Portfolio theory. It has become standard practice in asset management to use an optimal portfolio to undertake investment strategies. The optimal portfolio for any particular investor is the portfolio on the efficient frontier that is tangent to the "utility curve" that defines that investor's relative risk aversion (Kihlstrom,1981). In his landmark paper, Markowitz suggested that it was essential to consider both risk and return when making an investment decision. Furthermore, if an investor invests in many shares they will achieve diversification and will result in a portfolio with lower risk. He proposed the use of mean variance portfolio optimization to generate an efficient frontier. Ruppert (2004: 143) argues that a portfolio is efficient if, for a given level of risk, it has maximum return and for a given level of return, it has minimum risk. The inputs to Markowitz's formulation are the expected return and the covariance structure of the portfolio. Markowitz showed how the optimal portfolio problem can be solved using quadratic programming. The Markowitz problem for computing an optimal portfolio using quadratic programming is stated as follows:

$$\begin{aligned} Min\ \sigma_p^2 &= \mathbf{W}'\mathbf{\Sigma}\mathbf{W} \text{ subject to} \qquad (1.1) \\ \mu_p &= E_k \\ \mathbf{W}'\mathbf{1}_p &= 1 \\ 0 &\leq w_i \leq 1, \forall i = 1, \ldots, p \end{aligned}$$

- $\mathbf{W}$ is a vector of weights, w_i $i = 1, \ldots, p$, or the proportion invested in each share;

- $\mathbf{\Sigma}$ is a covariance matrix of share returns;
- σ_p^2 is a variance of a portfolio with p shares; and
- μ_p is the expected returns of a portfolio with p shares.

Solving Equation (1.1) for E_k results in optimal portfolios which lie on the efficient frontier and the portfolio manager's problem of selecting an optimal portfolio is reduced to selecting portfolios on the efficient frontier.

A large number of theorists have studied various ways of altering the inputs to improve the resulting optimal portfolio. William Sharpe (1970) uses index models to estimate the inputs into the Markowitz formulation. One advantage of the Sharpe index model is that it can be extended to dynamic time series regression models. Troskie et al. (2008) amends the time series models by changing the covariance structure of the share returns. This will be referred to as the TrosHos model throughout the thesis. Dynamic time series models make many assumptions that affect the resulting optimal portfolio. Sharpe (1970) presented a linear regression model which he termed the Sharpe Single Index model and is formulated as follows:

$$\begin{aligned} R_{it} &= \alpha_i + \beta_i I_t + e_{it} \qquad (1.2) \\ E(e_{it}^2) &= \sigma_{ei}^2 \\ E(e_{it}e_{is}) &= 0, \forall t \neq s = 1, \ldots, N, \\ E(e_{it}I_t) &= 0, \forall t = 1, \ldots N, \\ E(e_{it}e_{jt}) &= 0, \forall t = 1, \ldots, N,\ i, j = 1, \ldots, p,\ i \neq j. \end{aligned}$$

- R_{it} is the log return from the i^{th} share at time t.
- I_t is the return on the market portfolio at time t.
- e_{it} is the residual term from regressing R_{it} against I_t.

- β_i is the slope coefficient of the i^{th} share.
- α_i is the is the intercept of the i^{th} share.

The TrosHos model assumptions are:

$$\begin{aligned}
R_{it} &= \alpha_i + \beta_i I_t + e_{it} \ , \quad i = 1, \ldots, p \ ; \quad t = 1, \ldots, N \\
E(e_{it}^2) &= \sigma_{ei}^2 = \sigma_i^2 = \sigma_{ii}, \quad i = 1, \ldots, p, \\
E(e_{it}e_{is}) &= 0, \forall t \neq s = 1, \ldots, N, \\
E(e_{it}I_t) &= 0, \forall t = 1, \ldots N, \\
E(e_{it}e_{jt}) &= \sigma_{ij}, \forall t = 1, \ldots, N, i, j = 1, \ldots, p, \ i \neq j.
\end{aligned} \tag{1.3}$$

From the above formulation of the Sharpe Single index model, it can be seen that the residuals (e_{it} and e_{is}) are assumed to be uncorrelated, whereas in the TrosHos model the residuals (e_{it} and e_{is}) are assumed to be correlated. Using the Sharpe and TroHos model formulations, the risk of a portfolio can be divided in two parts namely: the risk of being invested in the market (which is represented by the portfolio's beta) and the unique risk or specific risk attributed to a particular share. A portfolio's risk can be calculated as follows:

$$var(R_p) = \begin{cases} \hat{\sigma}_I^2 \mathbf{W}' \hat{\boldsymbol{\beta}} \hat{\boldsymbol{\beta}}' \mathbf{W} + \mathbf{W}' \hat{\boldsymbol{\Omega}} \mathbf{W} \\ \text{Market Risk } + \text{Unique Risk} \end{cases} \tag{1.4}$$

$$Var(R_p) = \begin{cases} \hat{\sigma}_I^2 \mathbf{W}'_S \hat{\boldsymbol{\beta}}_S \hat{\boldsymbol{\beta}}'_{\mathbf{S}} \mathbf{W}_S + \sum_i^p w_{S(i)}^2 \hat{\sigma}_{S(i)}^2 & (Sharpe) \\ \hat{\sigma}_I^2 \mathbf{W}'_{TH} \hat{\boldsymbol{\beta}}_{TH} \hat{\boldsymbol{\beta}}'_{TH} \mathbf{W}_{TH} + \sum_i^p w_{TH(i)}^2 \hat{\sigma}_{TH(i)}^2 & (TrosHos) \\ + \sum_{i \neq j}^p w_{TH(i)} w_{TH(j)} \hat{\sigma}_{TH(ij)} & \end{cases}$$

Where:

$\hat{\sigma}_I^2$ = is variance vector of the portfolio
$\hat{\boldsymbol{\beta}}$ = beta coefficient vector of the portfolio
$\hat{\boldsymbol{\Omega}}$ = covariance structure of the portfolio

The last equations are the Sharpe and TrosHos formulations. The implications of these formulations will be explained in more detail during the development of the thesis. The assumptions of the TrosHos model will change the covariance structure of the portfolio and the resulting efficient frontier.

A further contribution to the Sharpe index model was the Capital Asset Pricing Model (CAPM) in which Sharpe used excess market returns to explain the excess return on a share.[1] This formulation is similar to the Sharpe index model, however, it has a number of assumptions which do not hold in the market.

The most important question is how these assumptions influence the portfolio manager's decision. These assumptions will have an impact on the efficient frontier which in turn will affect the resulting optimal portfolio. It is therefore important for the portfolio managers to test these assumptions. Troskie et al. (2008) found that for a selection of shares from the JSE the resulting residuals from the regression of the share returns against the market index (incorporate oveall index) returns are correlated, contrary to the assumptions of the Sharpe Index Model. In this thesis we will take this into account in the formulation of the CAPM into the TrosHos and Sharpe Index Model.

[1]Using the CAPM, it can be shown that $\mu_R - \mu_f = \beta_j \left(\mu_m - \mu_f\right)$."Excess expected return" means the amount by which the expected return on a portfolio exceeds the risk-free rate of return and is also called the risk premium. (Ruppert: 2004, 227)

1.2 Study aims and objectives

Using the Markowitz and Sharpe Single Index models results in different portfolios, due to the number of assumptions that these models impose. The thesis will aim to test some of the assumptions of the Sharpe single index model. The TrosHos model's efficient frontier and optimal portfolio is compared to the Sharpe Single Index model. The CAPM model assumptions are discussed and incorporated into the Sharpe and TrosHos models. Dynamic times series models are used to capture the serial autocorrelation and heteroskedaticity displayed by the residuals. The objectives of the thesis are:

- Investigate the Markowitz model as a basis for generating efficient frontiers and optimal portfolios;
- Compare the Markowitz model and its extension in the Sharpe Single index model and TrosHos model;
- Investigate dynamic time series models and how they can be used to extend the results of the Sharpe Single index models and the TrosHos models;
- Investigate the Capital Asset Pricing Model and compare the optimal portfolio of this model to the Sharpe Single index model and the TrosHos model;
- Adjust the Sharpe single index model and TrosHos model for the Capital Asset Pricing Model assumptions and compare the resulting optimal portfolios; and
- Incorporate dynamic time series models into the Sharpe CAPM and TrosHos CAPM.

1.3 Description of the data used in the investigation

The data used in the investigation is monthly share price data from the Johannesburg Stock Exchange (incorporate). This was obtained from Data Stream Advance, the online data bank and also from the incorporate Data Bank from the Statistical Sciences Department at the University of Cape Town. Whenever we plot the efficient frontier and optimal portfolio in the thesis we will use monthly expected return and monthly standard deviation values. The Data used is from January 1996 to April 2009. The data was converted into log monthly returns by using the following formula:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1.5}$$

where

- r_t is the return of the share at time t.
- p_t is the price of the share at time t.

The data period was selected bearing in mind that too short a period will not produce reliable results and too long a period will loose its relevance to the current time. Thus a period of twelve years was used and the data was checked to see if it was stationary. The log returns were found to be stationary hence it was possible to use the asymptotic results such as the expected return and variance of the shares. Specific risk can be diversified away by increasing the number of shares in a portfolio Ruppert (2004: 234). Given that we are investigating the specific risk of a portfolio we will restrict our portfolio to 9 shares

The Johannesburg inter bank agreed rate (JIBAR) was used as a proxy for the risk-free rate which is quoted as Nominal Annual Compounded Quarterly Rate (NACQ). This is also known as the 3 month JIBAR rate. From 1994 until 2001 the agreed bank rate was very volatile peaking at 21% *p.a.* in 1998 and reaching a low of 5% *p.a.* in 1995. The risk-free rate stabilized to 8% *p.a.* from 2004 to 2008. The decision on the appropriate risk-free rate to use is important. The rate used should be relevant to the period in which the analysis is being conducted. Using a risk-free rate which was applicable 15 years ago will give misleading results, hence we used a risk-free rate of 8% *p.a.* and transformed it into a monthly log rate to be consistent with the monthly log returns. The log monthly risk-free rate was 0.6%.

Table 1: Shares used in the Portfolio

Eviews Code	**Share**	**Sector**
r1	Anglo American	Mining
r2	Impala Platinum	Mining
r3	Pick and Pay	Food and Drug Retailers
r4	Remgro	Diversified Industrials
r5	Absa	Banks
r6	Richemont	Clothing and Footware
r7	Sasol	Oil and Gas Producers
r8	Tiger Brand	Food Processors
r9	Afrox	Chemicals

Table 1 displays the list of the 9 shares used in the investigation and the codes used in the Eviews programmes used in the thesis.

1.4 Scope and Limitations

The thesis uses a number of models which make use of data. The data has to satisfy a number of statistical assumptions including the underlying distribution, independence, correlation and heteroskedasticity. The models we used have extensive statistical and mathematical theory. The results of the analysis depends on the portfolio of shares chosen for the investigation. The number of shares used in the analysis influences the results of the investigation.We bear this in mind when composing the different optimal portfolios and analysing the results. This is not a theoretical study as it is an application piece and thus the development of new theory will not be explicitly addressed.

1.5 Plan of Development

In Chapter 2 we review the Markowitz model and determine the efficient frontier and optimal portfolios resulting from this model. This is an important chapter since it introduces the efficient frontier and the quadratic programming algorithm used to generate efficient frontiers.

Chapter 3 will introduce the Capital Asset Pricing Model. In introducing this model we will follow chapter 7 of "Statistics and Finance: An Introduction" by Ruppert (2004) closely. The Sharpe Single Index and TrosHos models will be investigated using formulation in Troskie et al. (2008). Finally we will incorporate the CAPM model into the Sharpe Single Index and TrosHos Model. These models have been termed the Sharpe CAPM and TrosHos CAPM. An empirical investigation of the Sharpe CAPM and TrosHos CAPM will conclude this Chapter.

In Chapter 4, dynamic time series models for the Sharpe CAPM and TrosHos CAPM are introduced. Again we compute and compare the efficient frontiers and optimal portfolios resulting from these models.

Chapter 5 gives a summary of the conclusions we have made in all previous chapters.

2 Markowitz Theory

2.1 Markowitz Theory

The origin of modern portfolio theory is a paper published in the Journal of Finance in 1956 authored by Harry Markowitz titled "Portfolio Selection". Markowitz's paper has formed a basis for most applications in the subject of Modern Portfolio Theory. In his paper, Markowitz used the covariance structure of the share returns as a proxy for the risk of a share or a portfolio. The Markowitz model is the first step in portfolio management; it assumes that a rational investor wants a high level of return for a given level of risk. Equivalently, the investor wants a lower level of risk for a given level of return.

2.2 Markowitz Formulation

In presenting Markowitz formulation we will use the same notation used in Troskie et al. (2008). In our portfolio we have p shares. Let the price of a share be P_t and the log return be defined as R_t =log$(P_t)-$log(P_{t-1}). The vector of returns of shares can be written as $\mathbf{R} = (R_{1,...,}R_p)'$ with $E(\mathbf{R}) = \boldsymbol{\mu} = (\mu_1, ..., \mu_p)'$ and covariance matrix $\boldsymbol{\Sigma} = E(\mathbf{R}-\boldsymbol{\mu})(\mathbf{R}-\boldsymbol{\mu})'$. At present we do not need the assumption of normality, but when the need arises we will in addition assume that $\mathbf{R} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ which is a multivariate normal distribution. A portfolio is an investment in shares, with weight w_i in each share. We can assume that w_i is a proportion of the investment available to the investor, such that $\sum_{i=1}^{p} w_i = 1$ and $0 \leq w_i \leq 1, \forall i$. Let $\mathbf{W} = (w_i, ..., w_p)'$ such that the portfolio return is defined as $\mathbf{R}_p = \mathbf{W}'\mathbf{R} = \sum_{i=1}^{p} w_i R_i$. The expected return is defined as $E(\mathbf{R}_p) = \mathbf{W}'E(\mathbf{R}_p) = \mathbf{W}'\mu = \sum_{i=1}^{p} w_i\mu_i = \mu_p$ and the variance is defined as $Var(\mathbf{R}_p) = \mathbf{W}'\boldsymbol{\Sigma}\mathbf{W}$.

If in addition $\mathbf{R} \backsim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ then $\mathbf{R}_p \backsim N(\mu_p, \sigma_p)$. By changing the weights w_i we can change the composition of the portfolio and its expected return.

Clearly we want to choose the weights so as to provide an investor with an expected return as large as possible $E(\mathbf{R}_p) = \mu_p$. Thus, an investor would want to maximize the expected return $E(\mathbf{R}_p) = \mu_p$. This will not be of much use if the variance of the portfolio σ_p is also large. The larger the variance the larger the risk and the reverse holds. Thus, an investor would want to choose the weights w_i such that the expected return $E(\mathbf{R}_p) = \mu_p$ is maximal, and at the same time the risk or variance σ_p^2 is minimal.

The portfolio problem is then $\max_{w_i} E(\mathbf{R}_p) = \mathbf{W}'\boldsymbol{\mu}_p$ subject to $\sum_{i=1}^{p} w_i = 1$.

2.3 Efficient Frontier

Ruppert (2004: 143), explains that within the Mean-Variance space, a portfolio is efficient if:

1. For a given amount of risk, the expected return is maximized, and;

2. For a given amount of return the risk is minimized.

According to Sharpe (1971) an efficient frontier can be computed by solving a non-linear QP quadratic programme. Figure 1 provides an illustration of an efficient frontier.

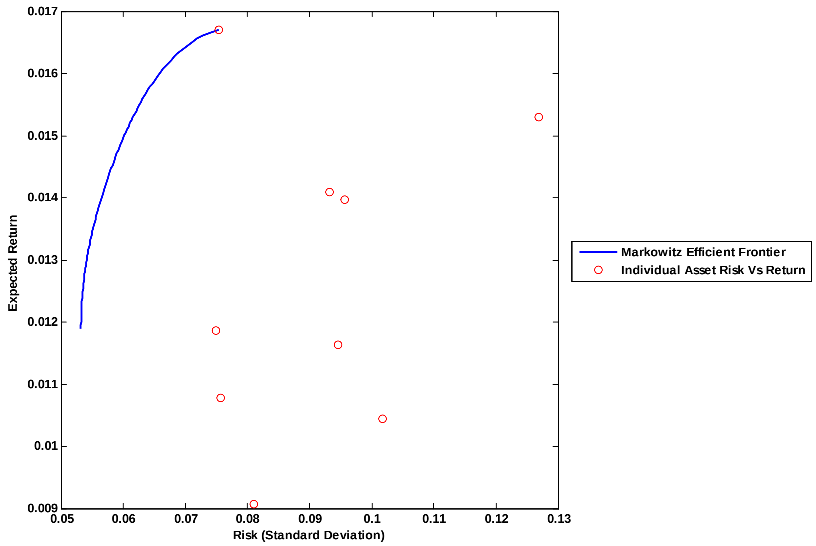

Figure1: Markowitz Efficient Frontier

Any portfolio lying on the blue line is a combination of the individual shares displayed in red circles and is said to be efficient. The different portfolios on this line will have different weights ($0 \leq w_i \leq 1, i = 1, \ldots, p$) of the shares. However, the individual shares not lying on the blue line are not efficient.

2.4 Capital Allocation Line and Tangency Portfolio

Tobin (1958) considered adding a risk-free[2] asset to the optimal portfolio. The addition of a risk-free asset resulted in portfolios which out-performed portfolios on the efficient frontier. The Capital Allocation Line (CAL) is a line of expected return plotted against risk that connects portfolios that can be formed using a risky portfolio and a risk-free asset. Elton et al. (2003: 86) proves that this is a straight line and is defined by the following equation:

[2]A risk- free Asset is an aseet with zero risk of default, such as a goverment bond or index linked goverment bond. ActEd Notes (2010)

$$E(r_c) = r_f + \sigma_c \left(\frac{E(r_p) - r_f}{\sigma_p} \right) \quad (2.12)$$

where

- p is the risky portfolio.
- f is the risk-free asset.
- c is a portfolio which consist of a combination of portfolios p and f.

Combining the market portfolio[3] with the risk-free asset, we get the Capital Market Line (CML) (Ruppert, 2004: 227). Portfolios on the CML have the highest Sharpe ratios[4] thus reflecting a higher risk return profile compared to any other portfolio on the efficient frontier. The CAL is a straight line from the risk-free rate to any feasible risky share portfolio, while the CML is a particular case of the CAL where the risky share portfolio in question is the market (tangency) portfolio. The CML equation is defined as (Ruppert, 2004: 227) :

$$E\left(r_c\right) = r_f + \sigma_c \left(\frac{E\left(r_m\right) - r_f}{\sigma_m} \right) \quad (2.13)$$

where

[3] A market portfolio contains all the securities and the weights of these securities are in proportion to their market values. It is a theoretical portfolio in which every available security is included at a level proportional to its theoretical market value. Fuller and Farrel (1987: 494)

[4] Sharpe Ratio can be thought of as "reward to-risk" ratio. It is the measure of excess return per unit of risk. Ruppert (2004: 143)

- m is the market portfolio.
- f is the risk-free asset.
- c is a portfolio which consist of a combination of portfolios m and f.

  In the following section we discuss some of the limitations of Markowitz formulation.

2.5 Limitation of the Markowitz formulation

Literature has been published on the limitations of the Markowitz formulation and below we highlight a few of the known limitations. The Markowitz optimization is very sensitive to errors in the estimates of the inputs. Chopra and Ziemba (1993) reveal that small changes in the input parameters can result in large changes in the composition of the optimal portfolio. Best and Grauer (1991) present empirical and theoretical results on the sensitivity of optimal portfolios to changes in expected returns. Chopra et al. (1993) reveals that using forecasts that do not accurately reflect the relative expected returns of different shares can substantially degrade the performance of the Markowitz formulation. We bear this in mind when composing the different optimal portfolios using the Markowitz formulation.

2.6 Computing the efficient frontier using Markowitz formulation

Our objective is to compute the efficient frontier for a selected portfolio of shares based on the Markowitz formulation. By adding the risk-free asset, we will determine the Capital Market Line and the tangency portfolio. This will be compared with the efficient frontiers and tangency portfolios generated from the Sharpe single index model, Sharpe CAPM, TrosHos and TrosHos CAPM model which we develop in later chapters.

2.7 Methodology

The mean and covariance structure of the share price returns were computed using Eviews 3. Matlab (7.6.0) was used for the optimization that is needed to develop the efficient frontier in Markowitz formulation. The mean and covariance structure of the share returns are the inputs to the optimization program. The Matlab program in appendix 3 was used to determine the optimal portfolio. Additional knowledge of the risk-free rate is required to successfully compute the optimal portfolio. We used the unbiased estimate of the covariance of returns from historic share returns data.

2.8 Primary Findings

Figure 2 displays the optimal efficient frontier from the above mentioned data set. The composition of the optimal portfolio is displayed in Table 2. Every point on the efficient frontier is a portfolio which gives a maximum return for a given level of risk or a minimum level of risk for a given level of return. The optimal portfolio (with the highest Sharpe ratio[5]) is the portfolio that an investor with a cost of capital equal to the risk-free rate of 8% *p.a.* will invest in. The amount that the investor will invest in the optimal portfolio and the risk-free asset will depend on their risk appetite. The expected return of the optimal portfolio is 1.6% per month and the standard deviation of the return is 6.5% per month.

[5] Sharpe's Ratio can be thought of as a "reward to-risk' ratio. It is the ratio of the reward quantified by the "excess expected return" to the risk as measured by the standard deviation.Ruppert (2004: 143)

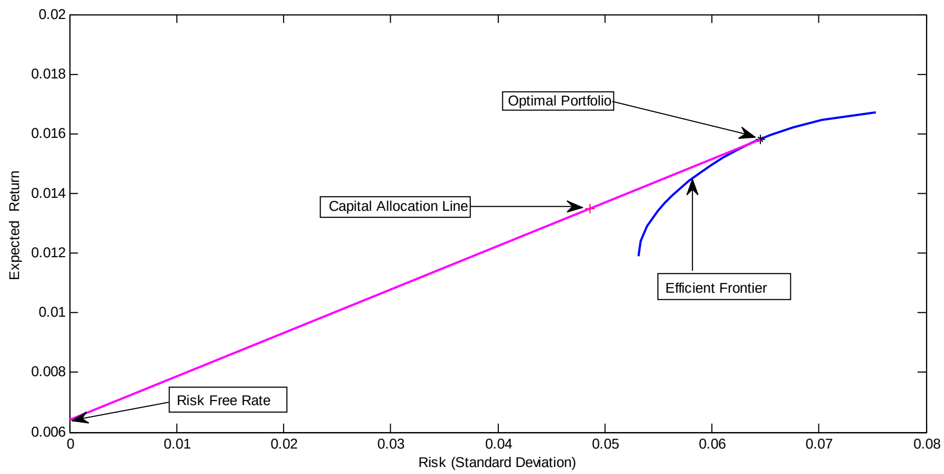

Figure 2: The Capital Allocation Line and the Optimal Portfolio

Table 2: Composition of Markowitz Optimal Portfolio

Share	**Proportion %**
Impala Platinum	10.8
Pick and Pay	4.0
Remgro	65.2
Absa	5.4
Sasol	14.7

The combination of shares in Table 2 constitutes the optimal portfolio. This portfolio maximizes return for any given risk and it minimizes risk for any given return. The optimal portfolio for the Markowitz formulation uses 5 out of the 9 available shares.

3 The Capital Asset Pricing Model (CAPM)

3.1 Introduction

Treynor (1961), Sharpe (1964), Lintner (1965a,b) and Mossin (1966) further developed the work done by Markowitz (1952) to develop the CAPM. The CAPM provides a relationship between the price of a share and its risk (Hageun, 2001: 201). Bodie et al.(1996: 236) state that the CAPM is useful in that it provides a benchmark rate of return when evaluating possible investments and helps in making an educated guess on the return of shares not yet traded. The CAPM is based on a number of assumptions[6]. Various authors including Jensen (1972) and Black (1972) have varied the assumptions of the CAPM. Farrel (1997: 55) states that the CAPM builds on the Markowitz model. He further argues that the Markowitz model is a normative model which gives an idea of how markets are supposed to behave, not how they actually behave. Given that the markets behave in the way that Markowitz stipulates, the CAPM determines the implications of:

- the behaviour of a share price;
- the sort of risk return relationship that one would expect; and
- the appropriate measure of risk for shares

Farrel (1997: 56) describes capital market theory in two main concepts namely: the Capital Market Line and the Security Market Line. In practice the Security Market Line and CAPM are used interchangeably. The difference between the two concepts is subtle but important. The Capital Market Line operates at the portfolio level and the Security Market line operates at the individual share level. The Capital Market line provides the relationship

[6] A list of the assumptions made by the CAPM. Bodie et al. (1996, 237)

between the expected return and risk for a portfolio of shares. While the Security Market Line provides the foundation for determining the relationship between the expected return and risk of individual shares. Additionally they give an idea of the appropriate measure of risk for a portfolio of shares and individual shares.

The chapter begins by investigating the Security Market Line and Formulations of the Capital Asset Pricing model. The layout of the first section follows Ruppert(2004: 232) closely. It then introduces the TrosHos and Sharpe single index model using notation from Troskie et al.(2008). Eventually we incorporate the Capital Asset Pricing Model formulation into the TrosHos model and Sharpe Single index model which we term the TrosHos CAPM and Sharpe CAPM. The final sections will investigate the empirical results of the resulting TrosHos CAPM and Sharpe CAPM models. All diagrams and results are computed using data specified in Chapter 2.

3.2 The Security Market Line and Formulation of CAPM

Ruppert (2004: 232) states that the Security Characteristic Line (SCL) is a regression model given by:

$$R_{j,t} = \mu_{f,t} + \beta_j \left(R_{m,t} - \mu_{f,t}\right) + \varepsilon_{j,t} \tag{3.1}$$

where

- $\varepsilon_{j,t} \sim N\left(0, \sigma^2_{\varepsilon,t}\right)$.
- $R_{j,t}$ is the return at time t of the j^{th} security.
- $R_{m,t}$ is the return at time t of the market portfolio.
- $\mu_{f,t}$ is the risk-free rate of return at time t.

- $\varepsilon_{j,t}$ is the error term of the j^{th} security at time t.
- $j = 1, ..., p$ and $t = 1, ..., N$.

The CAPM assumes that the $\varepsilon_{j,t}$ are uncorrelated across all shares. That is, $\varepsilon_{j,t}$ is uncorrelated with $\varepsilon_{j',t}$ for $j \neq j'$. Part of our empirical investigation is to assess the impact that this assumption has on the generation of the efficient frontier and the optimal portfolio. The implication of this assumption is that the error terms of the different shares are uncorrelated. The TrosHos model assumes that the error terms are correlated. Using this assumption generates a different efficient frontier from that generated when using the Markowitz model and Sharpe Single index model. As a result the optimal portfolios will be different for the different models.

Ruppert (2004: 232) applies the expectations operator to equation 3.1 to get the Security Market Line (SML).

$$E(R_{j,t}) = \mu_{j,t} = \mu_{f,t} + \beta_j \left(\mu_{m,t} - \mu_{f,t}\right) \tag{3.2}$$

where

- $\mu_{j,t} = E(R_{j,t})$ $\forall j$ and $\forall t$.
- $\mu_{m,t} = E(R_{m,t})$ $\forall m$ and $\forall t$.

Equation 3.2 is called the SML. It is important to note that the SML gives information on returns but not on the covariance of the returns. To get information on the covariance of returns we need to use the SCL. From the SCL we can deduce that

$$\begin{aligned} Var(R_{j,t}) &= \sigma_j^2 = \beta_j^2 \left(\sigma_m^2\right) + \sigma_{\varepsilon,j}^2 \\ \text{and } \sigma_{jj'} &= \beta_{jj'} \left(\sigma_m^2\right) \text{ for } j \neq j' \\ \text{and that } \sigma_{m,j} &= \beta_j \left(\sigma_m^2\right) \end{aligned} \tag{3.3}$$

The total risk of the j^{th} share is thus

$$\sigma_j = \sqrt{\beta_j^2 \left(\sigma_m^2\right) + \sigma_{\varepsilon,j}^2} \tag{3.4}$$

Which has two components:

1. $\beta_j^2 \left(\sigma_m^2\right)$ is called the market or systematic component of risk.

2. $\sigma_{\varepsilon,j}^2$ is called the unique, non market or unsystematic component of risk.

3.3 Computing the optimal efficient frontier and optimal portfolio of the CAPM

In our investigation we aim to determine, the expected excess returns and covariance structure implied by the CAPM. We then compute the efficient frontier using quadratic programming. Using the risk-free rate of 8% *p.a.*, we determine the optimal portfolio for the given set of shares. As we proceed with the thesis, we will compare the efficient frontier generated using the TrosHos model and Sharpe single index model. Both these models will be adjusted for the CAPM assumptions.

3.4 Description of data used in the investigation

For consistency we will use the same data set used in Chapter 2. The incorporate Overall Index will be included as the market portfolio over the same period. According to the CAPM model, we regress the excess security returns against the excess market returns.

3.5 Method of Investigation

The initial step is to compute the monthly log returns of each share and the monthly log return of the market portfolio. We then determine the excess of the monthly log returns over the log risk-free rate for each security. To achieve this we use data for the log risk-free rate from January 1996 to March 2009. To determine the expected return we regress the excess monthly log returns of the shares $\left(R_{j,t} - \mu_{f,t}\right) = R^*_{j,t}$ on the excess monthly log return of the incorporate overall index $\left(R_{m,t} - \mu_{f,t}\right) = R^*_{m,t}$. We now calculate the alpha and beta of each of the shares. We will use $R^*_{j,t} = \alpha_j + \beta_j \left(R^*_{m,t}\right) + \varepsilon_{j,t}$ to determine the estimated return of each share. All calculations are done on Eviews 3 including the computation of the covariance structure. The results are used as inputs in a Matlab program to compute the efficient frontier and optimal portfolio using the risk-free rate of 8% *p.a.* In all cases to follow, will use the above method to determine efficient frontiers and optimal portfolio when using CAPM assumptions.

3.6 Primary Findings

Figure 3 displays the efficient frontier obtained using the methodology we have just described above. The composition of the optimal portfolio is given in Table 3. The optimal portfolio was determined using the risk-free rate of 8% *p.a.* The estimated return of the optimal portfolio is (1.6%) per month and the standard deviation of the returns is (6.5%) per month.

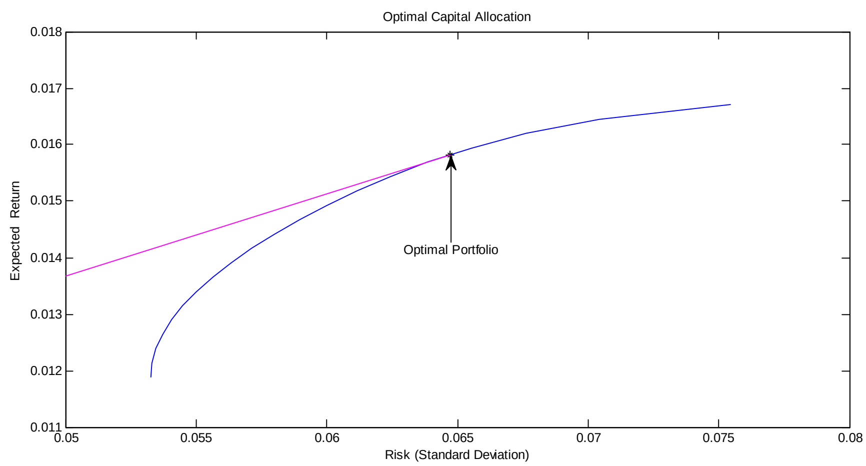

Figure 3: CAPM Efficient Frontier and Optimal Portfolio

Table 3: Composition of the CAPM Optimal Portfolio

Share	**Proportion %**
Impala Platinum	10.8
Pick and Pay	3.9
Remgro	65.6
ABSA	5.3
Sasol	14.4

The portfolio of shares in Table 3 constitutes the optimal portfolio. It maximizes the return for a given amount of risk and it minimizes risk for a given amount of return. The CAPM optimal portfolio composition is similar to the Markowitz optimal portfolio in chapter 2.

3.7 Introducing the Sharpe and TrosHos Models

The Sharpe Single Index Model, as amended by Hossain et al. (2005), is formulated as

$$R_{it} = \alpha_i + \beta_i I_t + e_{it}\ ,\ \ i = 1, \ldots, p\ ;\ \ t = 1, \ldots, N, \tag{3.5}$$

$$\begin{aligned}
E(e_{it}^2) &= \sigma_{ei}^2 = \sigma_i^2 = \sigma_{ii}, \quad i = 1, \ldots, p, \\
E(e_{it}e_{is}) &= 0, \ \ t \neq s = 1, \ldots, N, \\
E(e_{it}I_t) &= 0, \ \ t = 1, \ldots N, \\
E(e_{it}e_{jt}) &= 0, \ \ t = 1, \ldots, N, \ \text{(Sharpe)} \qquad (3.6) \\
E(e_{it}e_{jt}) &= \sigma_{ij}, \ \ t = 1, \ldots, N, \ \text{(TrosHos)} \qquad (3.7) \\
i, j &= 1, \ldots, p, \ i \neq j.
\end{aligned}$$

The Sharpe model in vector notation is given as

$$\mathbf{R}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{I} + \mathbf{e_t}\ ,\ \ t = 1, \ldots, N, \tag{3.8}$$

where

$$\mathbf{R}_t = \begin{pmatrix} R_{1t} \\ \vdots \\ R_{pt} \end{pmatrix}, \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_p \end{pmatrix} \boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_p \end{pmatrix} \text{ and } \ \mathbf{e}_t = \begin{pmatrix} e_{1t} \\ \vdots \\ e_{pt} \end{pmatrix} \tag{3.9}$$

so that (conveniently dropping the index t)

$$E(\mathbf{R}) = \boldsymbol{\alpha} + \boldsymbol{\beta}\mu_I \tag{3.10}$$

and

$$cov(\mathbf{e}) = \begin{pmatrix} \sigma_{e1}^2 & 0 & \dots & 0 \\ 0 & \sigma_{e2}^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \sigma_{ep}^2 \end{pmatrix} \tag{3.11}$$

which implies that

$$cov(\mathbf{R}) = \sigma_I^2 \boldsymbol{\beta}\boldsymbol{\beta}' + \begin{pmatrix} \sigma_{e1}^2 & 0 & \dots & 0 \\ 0 & \sigma_{e2}^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \sigma_{eq}^2 \end{pmatrix}. \tag{3.12}$$

Where:

μ_I = expected return of the index.

Equation 3.6 displays Sharpe's assumption that the residuals are uncorrelated whereas equation 3.7 displays the assumptions of the TrosHos Model.

For portfolio $R_p = \mathbf{W}'\mathbf{R}$ we have

$$E(\mathbf{R}_p) = \mathbf{W}'(\boldsymbol{\alpha} + \boldsymbol{\beta}\mu_I) = \mu_p \tag{3.13}$$

and the risk

$$var(R_p) = \begin{cases} \hat{\sigma}_I^2 \mathbf{W}'\hat{\boldsymbol{\beta}}\hat{\boldsymbol{\beta}}'\mathbf{W} + \mathbf{W}'\hat{\boldsymbol{\Omega}}\mathbf{W} \\ \text{Market Risk } + \text{Unique Risk} \end{cases} \tag{3.14}$$

$$Var(R_p) = \begin{cases} \hat{\sigma}_I^2 \mathbf{W}'_S \hat{\boldsymbol{\beta}}_S \hat{\boldsymbol{\beta}}'_{\mathbf{S}} \mathbf{W}_S + \sum_i^p w_{S(i)}^2 \hat{\sigma}_{S(i)}^2 & (Sharpe) \\ \hat{\sigma}_I^2 \mathbf{W}'_{TH} \hat{\boldsymbol{\beta}}_{TH} \hat{\boldsymbol{\beta}}'_{TH} \mathbf{W}_{TH} + \sum_i^p w_{TH(i)}^2 \hat{\sigma}_{TH(i)}^2 & (TrosHos) \\ + \sum_{i \neq j}^p w_{TH(i)} w_{TH(j)} \hat{\sigma}_{TH(ij)} & \end{cases}$$

Where:

$\hat{\sigma}_I^2 =$ is variance vector of the portfolio
$\hat{\boldsymbol{\beta}} =$ beta coefficient vector of the portfolio
$\hat{\boldsymbol{\Omega}} =$ covariance structure of the portfolio

The term $\mathbf{W}'\boldsymbol{\Omega}\mathbf{W}$ in the variance of $\mathbf{R}_p$, plays an important role in the formulation of the risk of a portfolio. The absence of the covariance between the stock residuals results in the loss of information under the Sharpe index model.

The term $\mathbf{W}'\boldsymbol{\Omega}\mathbf{W}$ is largely responsible for the difference between the Sharpe Index Model and the TrosHos model. The differences are magnified if all the covariances (correlations) are either positive or negative. If all the covariances (correlations) are positive then the Sharpe model underestimates the risk and if all the covariances (correlations) are negative then the Sharpe model overestimates the risk.

3.8 Introducing the Sharpe CAPM and TrosHos CAPM

The next step is to adjust the Sharpe Single model and TrosHos model for the assumptions of the Capital Asset Pricing Model. We use the excess log returns of the shares and excess log returns of the incorporate overall index $R_{it} - R_{ft} = \alpha_i + \beta_i (I_t - R_{ft}) + e_{it}\ i = 1, \ldots, p\ ; \quad t = 1, \ldots, N$. To get the expected returns we will regress the shares excess returns on the incorporate overall excess returns. We call the resulting models the Sharpe CAPM model and the TrosHos CAPM model. Henceforth when we refer to the Sharpe model we will be referring to the Sharpe Single Index CAPM model and when we refer to the TrosHos model we will be referring to the TrosHos CAPM model.

$$
\begin{aligned}
R_{it} - R_{ft} &= \alpha_i + \beta_i \left(I_t - R_{ft} \right) + e_{it}, i = 1, \ldots, p \; ; t \; = 1, \ldots, N, \\
R_{it}^* &= \alpha_i + \beta_i I_t^* + e_{it} \; , \;\; i = 1, \ldots, p \; ; \;\; t = 1, \ldots, N \qquad (3.15)
\end{aligned}
$$

Using the expected returns and covariance structure we can then generate the efficient frontier. A risk-free rate of 8% *p.a.* is used to determine the optimal portfolio resulting from the Sharpe CAPM and TrosHos CAPM model.

3.9 Investigation into the efficient frontier and optimal portfolios of the Sharpe CAPM and TrosHos CAPM

Firstly we will compute the efficient frontier using the Markowitz, Sharpe single index and the TrosHos models. Next we compute the efficient frontier for the Sharpe CAPM and the TrosHos CAPM formulations. The reason for computing the Markowitz efficient frontier is for comparison purposes. Finally we will determine the implications to the portfolio manager by comparing the composition of the optimal portfolios and the shifting of the efficient frontier resulting from the three models. The risk-free rate of 8% *p.a.* is used in determining the optimal portfolio. The remainder of the thesis will be comparing the Sharpe CAPM and TrosHos CAPM with each other.

3.10 Methodology

For the Sharpe CAPM and the TrosHos CAPM, the same methodology used to compute the CAPM efficient frontier in the previous section is used. The same method as in chapter 2 was used to compute the Markowitz efficient frontier.

3.11 Description of data used in the investigation

The same data was used as in chapter 2. The incorporate Overall Index will be included as the market portfolio over the same period.

3.12 Analysis and Conclusion

Figure 4 displays the comparison between the different efficient frontiers. The Markowitz and the TrosHos CAPM efficient frontiers are very similar. The Markowitz efficient frontier is included for comparison purposes. Figure 4 and Table 4 compares the efficient frontiers and optimal portfolios of the: Markowitz, Sharpe and TrosHos models. Figure 5 and Table 5 compares the efficient frontiers and optimal portfolios of the: Markowitz, Sharpe CAPM and TrosHos CAPM models.

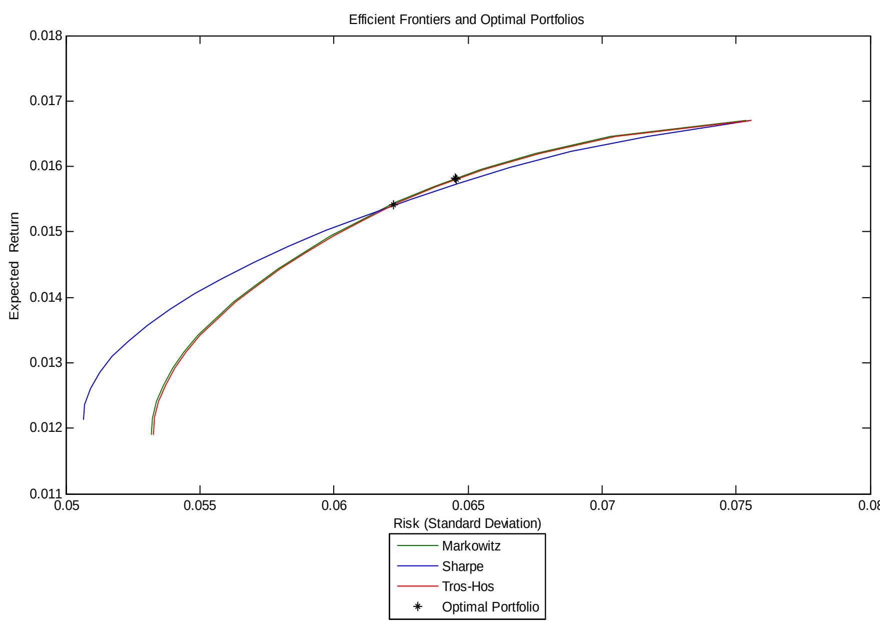

Figure 4: Sharpe and TrosHos Efficient Frontiers and Optimal Portfolios

Table 4: Comparison of the Markowitz, Sharpe and TrosHos Optimal Portfolios

Share	**Markowitz %**	**Sharpe %**	**TrosHos %**
Impala Platinum	10.8	0	10.8
Pick and Pay	4.0	10.2	4.1
Remgro	65.2	63.7	64.9
Absa	5.4	14.2	5.5
Richemont	0	4.9	0
Sasol	14.7	7.1	14.7
Expected Return-p.a.	19.0	18.5	19.0
Portfolio Variance-p.a.	22.3	20.7	22.4

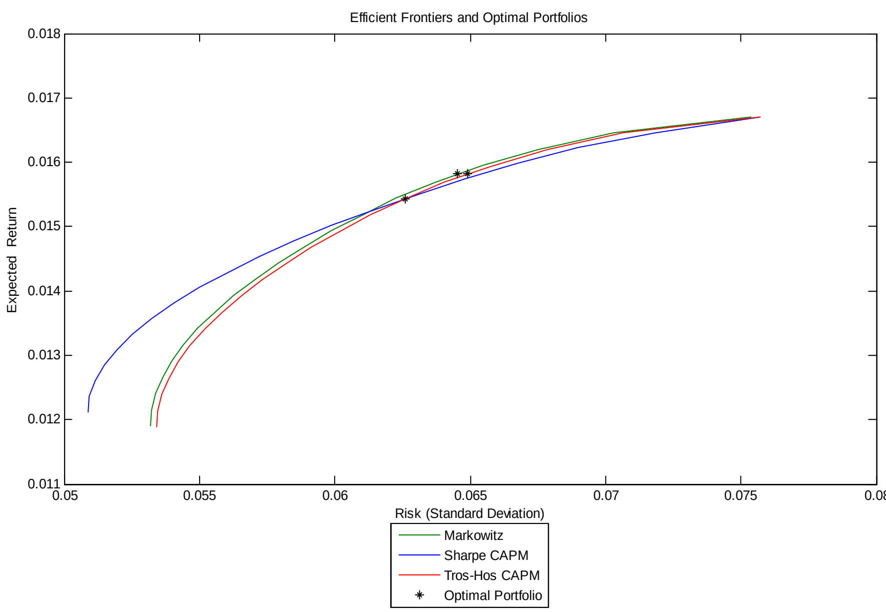

Figure 5: TrosHos CAPM and Sharpe CAPM Efficient Frontiers and Optimal Portfolios

Table 5: Comparison of the Sharpe CAPM and TrosHos CAPM Optimal Portfolios

Share	**Sharpe CAPM %**	**TrosHos CAPM %**
Impala Platinum	2.7	10.8
Pick and Pay	6.7	3.9
Remgro	63.1	65.5
Absa	13.8	5.4
Richemont	4.6	0
Sasol	7.2	14.5
Tigerbrand	1.9	0
Expected Return-p.a.	18.5	19.0
Portfolio Variance-p.a.	21.7	22.5

Best and Grauer (1991) show that portfolio composition is extremely sensitive to changes in the expected returns of shares. Since the TrosHos CAPM and Sharpe CAPM models make use of the same share expected returns but different covariance structures their portfolio composition can be compared. However, the Markowitz and CAPM models have different expected returns as inputs when computing the efficient frontiers. Therefore, Best and Grauer's findings suggest that the Markowitz and CAPM models cannot be compared.

Merton (1980) found that the estimates of variances and covariances are more accurate than the estimates of the expected returns. It makes the comparison of the TrosHos CAPM and Sharpe CAPM more sensible because the only difference between the two models is in the covariance structure. Although the expected returns of the Markowitz and TrosHos models are different the optimal portfolios generated by the models are very similar. The optimal portfolios of the Markowitz and TrosHos models are very different from the optimal portfolio given by the Sharpe CAPM model because the Markowitz and TrosHos CAPM model take account of the covariance of share expected returns. On the other hand, the Sharpe CAPM model does not take into account the covariance of the share expected returns.

Markowitz (1959) proved that positive covariances in share expected returns increases risk, while negative covariances reduces risk. The study displays that the TrosHos CAPM model for our given portfolio of shares contains more information than the Sharpe CAPM model. In Figure 4 and 5, for lower levels of risk, the TrosHos CAPM model is below the Sharpe CAPM model. When the risk levels increase the TrosHos Model shifts upwards and eventually crosses the Sharpe CAPM model. For higher levels of risk the TrosHos CAPM model lies above the Sharpe CAPM mode.

This study extends on the findings of Hossain et al. (2005) since we included CAPM formulation in the Sharpe and TrosHos model whereas they did not.

4 CAPM Dynamic Time Series Models Introduction

A major advantage of the Index model over the risk return Markowitz model is that it can be extended to include serial autocorrelation and heteroskedasticity that appear in the residuals of index models. Similar to Troskie et al.(2008), Mupambirei (2008) and Gilbert (2007) we will consider the effect of the residuals on the Index models. However in this thesis we will not consider the Sharpe or TrosHos models, we will consider the Sharpe CAPM and TrosHos CAPM models.

The Sharpe CAPM model is a regression model. The formulation of the Sharpe CAPM model is as follows:

$$
\begin{aligned}
R_{it} - R_{ft} &= \alpha_i + \beta_i \left(I_t - R_{ft}\right) + e_{it} \ , \forall i = 1, \ldots, p \ ; \quad t = 1, \ldots, N, \\
R_{it}^* &= \alpha_i + \beta_i I_t^* + e_{it} \ , \forall i = 1, ..., p \ ; \quad t = 1, \ldots, N, \\
E(e_{it}^2) &= \sigma_{ei}^2 = \sigma_i^2 = \sigma_{ii}, \forall i = 1, \ldots, p, && (4.1) \\
E(e_{it} e_{is}) &= 0, \forall t \neq s = 1, \ldots, N, && (4.2) \\
E(e_{it} I_t) &= 0, \forall t = 1, \ldots N, && (4.3) \\
E(e_{it} e_{jt}) &= 0, \forall t = 1, \ldots, N, \ \text{(Sharpe CAPM)} && (4.4) \\
\forall i, j &= 1, \ldots, p, \ i \neq j.
\end{aligned}
$$

The implications of the fourth assumption (4.4) on the efficient frontier and the optimal portfolio was examined in chapter 3. In this chapter we will study the impact that the first two assumptions (4.1 and 4.2) will have on the efficient frontier and optimal portfolio. The first two assumptions indicate that the residual terms have no serial autocorrelation and they are homoskedatic. If we combine the first two assumptions and the assumption that residuals are normally distributed then the Sharpe model satisfies the Gauss-Markov theorem (Ruppert, 2004: 170). However, Gilbert (2007) found that for some incorporate shares the Gauss-Markov theorem is not satisfied by the Sharpe model. Similarly for our given group of shares we found that the Sharpe CAPM model does not satisfy the Gauss-Markov theorem.

The Gauss-Markov theorem states that the estimates from the least squares method will be the best since they have the minimum variance in comparison to any other estimates. To use the Gauss-Markov theorem, we should ensure that the assumptions of the theorem hold, that the residuals are uncorrelated, have a mean of 0 and the variance is constant (homoscedastic). However, if the assumptions do not hold, then the least squares estimates can not be assumed to be the best. This indicates that we have to alter the model by including a model for the residuals. We start by considering a simple model for the residuals: the ARMA model, which takes autocorrelation and partial autocorrelation of the residuals into account. We then extend this by modelling the variance of the residuals. The GARCH(1,1) model captures most of the variance of the residuals. Finally we capture both the autocorrelation and variance of the residuals by using ARMA/GARCH models. We will introduce some basic concepts of time series before fitting time series models to the Sharpe CAPM and TrosHos CAPM models.

4.1 The Autoregressive Model

An autoregressive process of order p, $AR(p)$ is given by

$$e_t = c + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_p e_{t-p} + \nu_t \tag{4.2}$$

where e_t is the residual from our regression at time t and $E(\nu_t) = 0$, $E(\nu_t^2) = \sigma_v^2$, $E(\nu_t v_s) = 0$ and c is a constant.

4.2 The Moving Average Model

A moving average model of order q, $MA(q)$ is given by

$$v_t = c + \alpha_1 \nu_{t-1} + \alpha_2 \nu_{t-2} + \cdots + \alpha_q \nu_{t-q} \tag{4.3}$$

where e_t is the residual from our regression at time t and $E(\nu_t) = 0$, $E(\nu_t^2) = \sigma_v^2$, $E(\nu_t v_s) = 0$ and c is a constant.

4.3 The Autoregressive Moving Average Model

An autoregressive moving average model of order $p, q, ARMA(p, q)$ is given by

$$e_t = c + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_p e_{t-p} + \nu_t + \alpha_1 \nu_{t-1} + \alpha_2 \nu_{t-2} + \cdots + \alpha_q \nu_{t-q} \tag{4.4}$$

where e_t is the residual from our regression at time t and $E(\nu_t) = 0$, $E(\nu_t^2) = \sigma_v^2$, $E(\nu_t v_s) = 0$ and c is a constant.

4.4 Generalized Autoregressive Conditional Heteroskedatic Model

A Generalized Autoregressive Conditional Heteroscedastic model of order p, q, is given by

$$\begin{aligned} a_t &= e_t - \mu_t && (4.5) \\ a_t &= \epsilon_t \sigma_t && (4.6) \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{i=1}^{q} \beta_i^2 \sigma_{t-i}^2 && (4.7) \\ \epsilon_t &\sim N(0,1) && (4.8) \end{aligned}$$

where e_t is the residual from our regression at time t, μ_t and σ_t are the conditional mean and variance at time t such that a_t is the mean adjusted residual from our regression at time t.

The GARCH model is used to capture the conditional variance (heteroskedasticity) of the residuals in an index model.

4.5 AR (p)/GARCH (1,1) Models

In some cases the AR(p)/GARCH(1,1) model captured most of the autocorrelation and variance of the residuals of the index models. The model is formulated as follows:

$$e_t = c + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_p e_{t-p} + \nu_t \quad (4.9)$$

$$\begin{aligned} a_t &= e_t - \mu_t \\ a_t &= \epsilon_t \sigma_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1^2 \sigma_{t-1}^2 \\ \epsilon_t &\sim N(0,1) \end{aligned} \tag{4.10}$$

where e_t is the residual from our regression at time t ,$E(\nu_t) = 0$, $E(\nu_t^2) = \sigma_v^2$, $E(\nu_t v_s) = 0, c$ is a constant. μ_t and σ_t are the conditional mean and variance at time t such that a_t is the mean adjusted residual from our regression at time t.

4.6 Heteroskedasticity

One of the assumptions of the least squares model is that the expected value of the error terms when squared is constant at any given point in time. This assumption is termed homoskedasticity, (Engle: 2001). If the variances of the error terms are different at different points in time then the data is said to be heteroskedastic.

According to Nelson (1991), the most used model for heteroskedasticity in share returns is the ARCH and GARCH models introduced by Engle (1982) and Bollerslev (1986). By modelling the variances of the residuals through using the ARCH and GARCH model we are correcting the deficiencies of the least squares model (Engle: 2001).

Nelson (1991) further explains that, by setting the conditional variance equal to a constant plus a weighted average (with positive weight) of past squared residuals, GARCH models elegantly capture the volatility clustering in share returns first noted by Mandelbrot (1963). For the purpose of this thesis we will restrict ourselves to the GARCH(1,1) models. We use the Swartz information criterion to assess the adequacy of the GARCH models.

4.7 Study objectives

The Sharpe CAPM and TrosHos CAPM models are index models. In their basic form they are least squares models. For the purposes of this study we will focus on the single index model. Index models assume that the residuals:

- have no serial correlation.
- are homoskedastic.

The purpose of the study is to determine the effect these assumptions have on the efficient frontier and the optimal portfolio.

4.8 Methodology

4.8.1 Serial autocorrelation

The first step is to assess the level of serial autocorrelation in the resulting residuals from the index model. This is undertaken by using the Box-Jenkins[7] procedure. The cut off point for the sample autocorrelation and partial autocorrelation is $\pm\frac{2}{\sqrt{T}}$ where $T =$ sample size.

4.8.2 Heteroskedasticity

To determine the presence of heteroskedasticity in the residuals we examine the partial correlations and Q statistics of the squared residuals from the regression. The cut off point for the sample partial correlation is $\pm\frac{2}{\sqrt{T}}$ where

[7] "A step by step guide of the Box-Jenkins Procedure".(Chatfield, C. and Prothero, D. :1973)

$T =$ sample size. The presence of the autocorrelation in the squared residuals indicates that heteroskedasticity is present in the series examined.

4.8.3 Serial Autocorrelation and Heteroskedasticity

We use the combination of the two methods above to eliminate serial autocorrelation and heteroskedasticity in the residuals. We check the residuals for serial autocorrelation by assessing the sample partial autocorrelation and sample autocorrelation of the residuals. This will lead us into selecting the appropriate ARMA model before assessing the sample partial correlation of the squared residuals. If there is evidence of heteroskedasticity, we fit the GARCH(1,1) model.

4.9 TrosHos CAPM and Sharpe CAPM Single Index Dynamic Time Series Models

The final step is to use the Sharpe CAPM and TrosHos CAPM model to estimate the means and covariance structure of our portfolio of shares.

$$\hat{\Phi} = \hat{\boldsymbol{\sigma}}_I^2 \hat{\boldsymbol{\beta}} \hat{\boldsymbol{\beta}}' + \hat{\boldsymbol{\Omega}} \tag{4.11}$$

We estimate $\boldsymbol{\Omega}$ using:

$$\hat{\boldsymbol{\Omega}}_S = \begin{pmatrix} \hat{\boldsymbol{\sigma}}_{e_{1,1}} & 0 & \dots & 0 \\ 0 & \hat{\boldsymbol{\sigma}}_{e_{2,2}} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \hat{\boldsymbol{\sigma}}_{e_{p,N}} \end{pmatrix} \text{(Sharpe CAPM)} \tag{4.12}$$

$$\hat{\mathbf{\Omega}} = \begin{pmatrix} \hat{\boldsymbol{\sigma}}_{e_{1,1}} & \hat{\boldsymbol{\sigma}}_{e_{1,2}} & \dots & \hat{\boldsymbol{\sigma}}_{e_{1,N}} \\ \hat{\boldsymbol{\sigma}}_{e_{2,1}} & \hat{\boldsymbol{\sigma}}_{e_{2,2}} & \dots & \hat{\boldsymbol{\sigma}}_{eN} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\boldsymbol{\sigma}}_{e_{p,1}} & \dots & \dots & \hat{\boldsymbol{\sigma}}_{e_{p,N}} \end{pmatrix} \text{ (TrosHos CAPM)} \qquad (4.13)$$

As a final step we compute and compare the efficient frontier and optimal portfolios for the LS, AR, GARCH and AR-GARCH using the Sharpe CAPM and TrosHos CAPM formulations. We compare equivalent TrosHos CAPM and Sharpe CAPM efficient frontiers. As an example we will compare the Sharpe CAPM-AR efficient frontier to the TrosHos CAPM-AR efficient frontier. To account for the fact that investors are exposed to different risk-free rates we used two different rates namely: 8%p.a. and 3%p.a. risk-free rates.

4.10 Least Squares Model

The least squares model refers to the TrosHos CAPM and Sharpe CAPM models in their original forms. These models do not incorporate the time series models or heteroskedastic models. We will compare these models to the equivalent AR, GARCH and AR-GARCH models. In all of the figures used in this chapter "Sharpe" should be read as Sharpe CAPM and "TrosHos" should be read as TrosHos CAPM model. Figure 6 displays the efficient frontier from the least squares models, while Table 6 displays the portfolio composition of the least squares models.

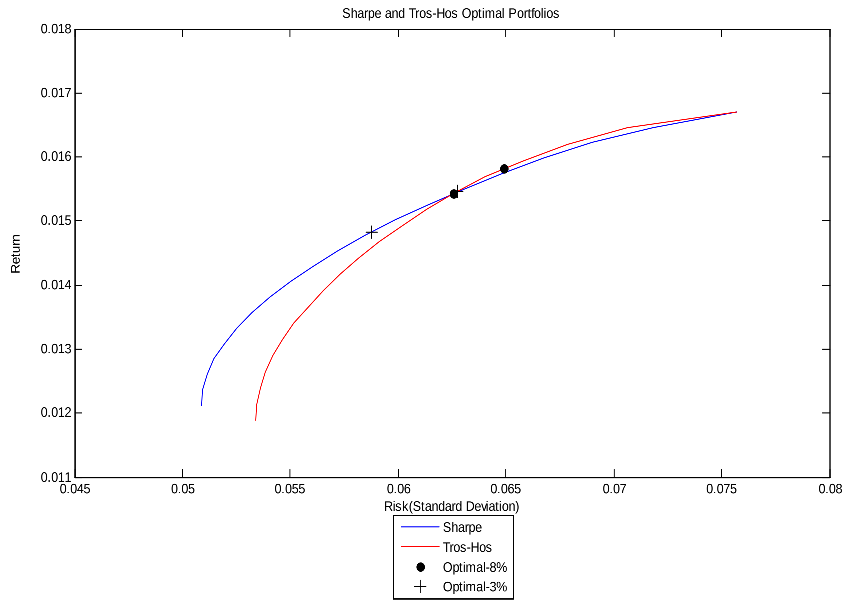

Figure 6: The Sharpe and TrosHos Optimal Portfolios

Table 6: Least squares optimal portfolio

Share	Sharpe 8%	Sharpe 3%	TrosHos 8%	TrosHos 3%
Anglo Ameriacan (incorporate)	0.0	0.0	0.0	0.0
Impala Platinum	2.8	1.1	10.8	10.9
Pick and Pay	6.7	8.9	3.9	9.0
Remgro	63.0	54.0	65.5	57.3
Absa	13.8	12.8	5.4	6.4
Richemont	4.6	9.0	0.0	0.6
Sasol	7.2	6.4	14.4	15.8
Tiger Brand	1.9	7.8	0.0	0.0
Afrox	0.0	0.0	0.0	0.0
Expected Return-p.a.	18.5	17.8	19.0	18.6
Portfolio St.Dev-p.a.	21.7	20.4	22.5	21.7

The Sharpe CAPM and TrosHos CAPM single index models assume that the residuals are uncorrelated, have an expected value of 0 and a constant variance. This indicates that it satisfies the Gauss-Markov theorem, hence its least squares estimates are Best Linear and Unbiased Estimates (BLUE). However, as we examined the data in the next session there is evidence that this is not the case. The (estimated) residuals are found to be correlated and do not have a constant variance. Thus, to ensure that the (estimated) residuals are uncorrelated we introduce the dynamic time series models to model the residuals.

4.11 Effects of Serial Autocorrelation

There is strong evidence of serial autocorrelation in 5 out of the 9 shares. There was no significant MA terms in any of the shares. We have documented the t-statistic of the significant AR terms in Table 7. Figure 6 displays the effect of serial autocorrelation on the efficient frontier. For low levels of risk in Figure 6 the TrosHos CAPM efficient frontier is below the equivalent Sharpe CAPM efficient frontier. As the levels of risk increase the TrosHos efficient frontier shifts upwards and crosses the Sharpe efficient frontier. For high levels of risk the TrosHos efficient frontier lies above the Sharpe efficient frontier.The TroHos CAPM efficient frontier contains more information than the Sharpe CAPM efficient frontier. By ignoring the covariances of the residuals some risk is ignored by the Sharpe CAPM efficient frontier. This is a common pattern with all of the efficient frontiers in subsequent figures.

In Table 8 we see the effect of serial autocorrelation on the optimal portfolios. The Least Squares optimal portfolio and the Sharpe optimal portfolio include the same shares but in different proportions. Appendix 1 list the adjusted R-Square, Swartz information criteria, residual variance, sample autocorrelation and sample partial correlation used when undertaking model selection.

Table 7: Regression statistics for the AR models

Share	AR-term	t-stat	AR-term	t-stat	AR-term	t-stat
Anglo*	-	-	-	-	-	-
Impala Platinum*	-	-	-	-	-	-
Pick and Pay	AR(1)	-3.96	AR(5)	-2.64	-	-
Remgro	AR(1)	-2.58	-	-	-	-
ABSA	AR(5)	-1.40	AR(6)	-1.56	-	-
Richemont*	-	-	-	-	-	-
Sasol*	-	-	-	-	-	-
Tiger Brand	AR(6)	-3.37	-	-	-	-
Afrox	AR(1)	-1.74	AR(3)	1.58	AR(4)	1.47
* No Significant Serial Autocorrelation						

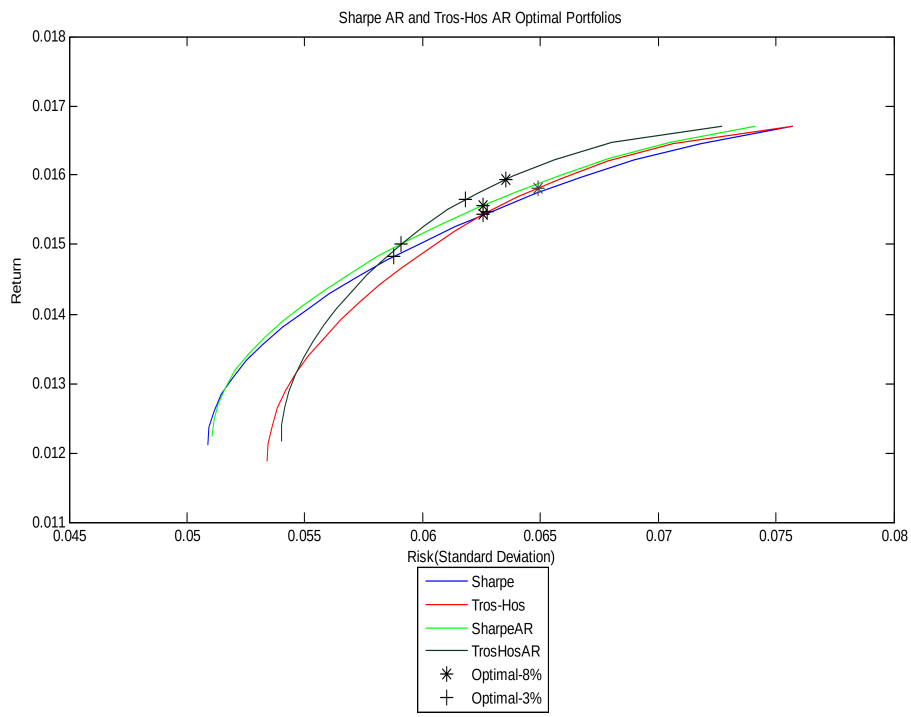

Figure 7: Sharpe AR and TrosHos AR

Table 8: Effects of autocorrelation on the optimal portfolios

Share	Sharpe	Sharpe	TrosHos	TrosHos
	Ar 8%	Ar 3%	Ar 8%	Ar 3%
Anglo	0.0	0.0	0.0	0.0
Impala Platinum	2.9	1.4	10.7	10.7
Pick and Pay	7.5	9.8	3.6	8.1
Remgro	66.1	57.6	69.9	63.3
Absa	12.4	11.5	3.2	4.0
Richemont	4.2	8.4	0.0	0.0
Sasol	6.9	6.2	12.6	13.9
Tiger Brand	0.0	5.1	0.0	0.0
Afrox	0.0	0.0	0.0	0.0
E(R)-p.a.	18.7	18.0	19.1	18.8
pt Std dev-p.a.	21.7	20.5	22.0	21.4

In the next section we will examine the effects that heteroskedasticity has on the efficient frontier and optimal portfolio.

4.12 Effects of Heteroskedasticity

Examining the tables of sample autocorrelation and partial autocorrelation of the squared residuals there was evidence of heteroskedasticity. We used the GARCH(1,1) to model the squared residuals. Table 9 displays the GARCH(1,1) model parameter estimates. Figure 7 displays the effects of heteroskedasticity on the efficient frontier. For lower levels of risk in Figure 7 the TrosHos efficient frontier is below the equivalent Sharpe efficient frontier. As the levels of risk increase the TrosHos efficient frontier shifts upwards and ends above the Sharpe efficient frontier.The TroHos CAPM efficient frontier contains more information than the Sharpe CAPM efficient frontier. By ignoring the covariances of the residuals some risk is ignored by the Sharpe CAPM efficient frontier The optimal portfolio of the least squares model is different to that of the GARCH model. Table 10 displays the effects of heteroskedasticity on the optimal portfolio. The GARCH optimal portfolios has a different composition to that of the least squares optimal portfolios.

In Table 10 we see the effect of heteroskedasticiy on the optimal portfolios. The Sharpe and TrosHos optimal portfolios have different compositions. Appendix 1 provides the adjusted R-Square, Swartz information criteria, residual variance, sample autocorrelation and sample partial correlation used when undertaking model selection.

Table 9: Regression Statistics for the GARCH model

Share	α_0	z statistic	α_1	z statistic	β_1	z-statistic
Anglo	0.0002	1.48	0.0981	2.01	0.8573	11.48
Impala Platinum*	-	-	-	-	-	-
Pick and Pay*	-	-	-	-	-	-
Remgro*	-	-	-	-	-	-
ABSA*	-	-	-	-	-	-
Richemont*	-	-	-	-	-	-
Sasol*	-	-	-	-	-	-
Tiger Brand*	-	-	-	-	-	-
Afrox	0.0006	1.92	0.1176	2.32	0.7636	8.52
* No Significant Heteroskedasticity						

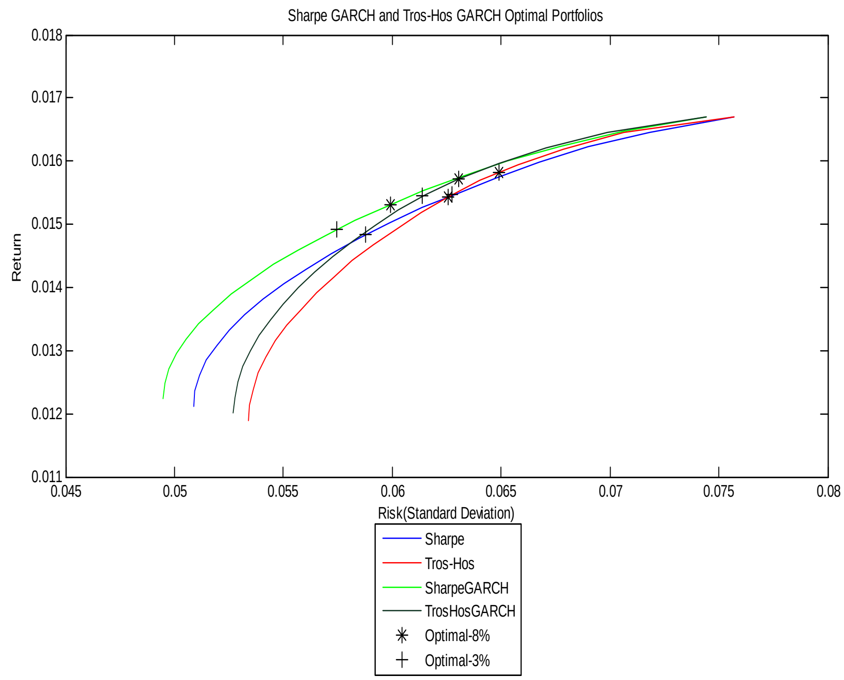

Figure 8: Sharpe GARCH and TrosHos GARCH

The next section will be examining the effects of autocorrelation and heteroskedasticity on the efficient frontier and optimal portfolio.

Table 10: Effects of heteroskedasticity on the optimal portfolio

Share	Sharpe	Sharpe	Tros-Hos	Tros-Hos
	Gar 8%	Gar 3%	Gar 8%	Gar 3%
Anglo	0.0	0.0	0.0	0.0
Impala Platinum	0.0	0.0	7.9	8.0
Pick and Pay	9.1	10.5	5.2	9.3
Remgro	61.4	55.0	64.2	57.8
Absa	16.0	15.1	7.1	7.9
Richemont	3.8	6.6	0.0	0.2
Sasol	7.8	7.0	15.6	16.8
Tiger Brand	1.9	5.8	0.0	0.0
Afrox	0.0	0.0	0.0	0.0
E(R)-p.a.	18.4	17.9	18.9	18.5
pt Std.Dev-p.a.	20.8	19.9	21.8	21.3

4.13 Effects of Serial Autocorrelation and Heteroskedasticity

Table 11 displays the AR-GARCH models fitted to residuals of the Index model. Figure 8 displays the effect that time series errors and heteroskedasticity have on the efficient frontier. For lower levels of risk the TrosHos CAPM efficient frontier is below the equivalent Sharpe CAPM efficient frontier. As the risk levels increase the TrosHos CAPM efficient frontier shifts upwards and it eventually crosses the Sharpe CAPM Efficient Frontier. For High levels of risk the TrosHos CAPM efficient frontier is above the Sharpe CAPM Efficient frontier.The TroHos CAPM efficient frontier contains more information than the Sharpe CAPM efficient frontier. By ignoring the covariances of the residuals some risk is ignored by the Sharpe CAPM efficient frontier Table 12 displays the optimal portfolio composition of the AR-GARCH models. The optimal portfolio of the AR-GARCH models have different shares compared to the optimal portfolio of the least squares model. But the optimal portfolio of the AR-GARCH and GARCH models are similar. The only difference is the proportion invested in each share.

In Table 12 we see the effect of heteroskedasticiy on the optimal portfolios. The Sharpe and TroHos Optimal portfolio have different compositions. Appendix 1 gives adjusted R-Square, Swartz information criteria, residual variance, sample autocorrelation and sample partial autocorrelation used in model selection.

Table 11: Significant AR and GARCH models

Share	**Autocorrelation**	**Heteroskedasticity**
Anglo	-	GARCH(1,1)
Impala Platinum*	-	-
Pick and Pay	AR(1), AR(5)	GARCH(1,1)
Remgro	AR(1)	GARCH(1,1)
ABSA	AR(6)	GARCH(1,1)
Richemont*	-	-
Sasol*	-	-
Tiger Brand	AR(6)	-
Afrox	AR(1),AR(3)	GARCH(1,1)
*No Serial Autocorrelation		

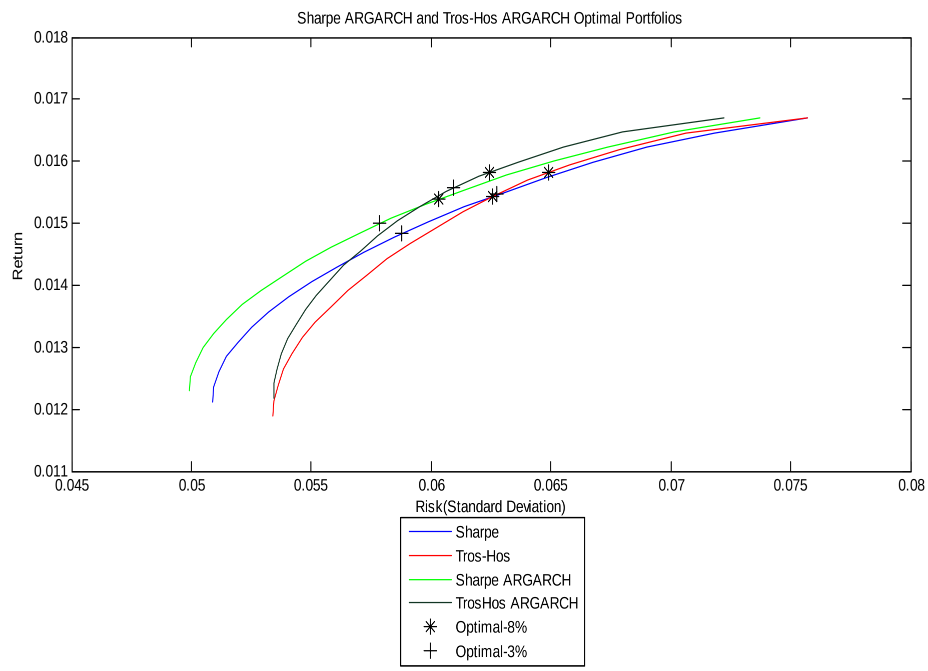

Figure 9: Sharpe AR-GARCH and TrosHos AR-GARCH

By modelling the autocorrelation and variance of the residuals results in the models satisfying the Gauss-Markov assumptions.

Table 12: Effects of Heteroskedasticity and Autocorrelation on the optimal portfolio

Share	Sharpe	Sharpe	TrosHos	TrosHos
	Ar-Gar8%	Ar-Gar3%	Ar-Gar8%	Ar-Gar3%
Anglo	0.0	0.0	0.0	0.0
Impala Platinum	0.0	0.0	7.9	8.0
Pick and Pay	9.9	11.6	5.1	9.0
Remgro	63.7	57.0	68.0	62.2
Absa	14.6	13.7	5.5	6.2
Richemont	4.3	7.4	0.0	0.0
Sasol	7.1	6.4	13.5	14.6
Tiger Brand	0.4	3.9	0.0	0.0
Afrox	0.0	0.0	0.0	0.0
E(R)-p.a.	18.5	18.0	19.0	18.7
pt Std.dev-p.a.	20.9	20.0	21.6	21.1

4.14 Conclusion

In this chapter we have incoporated the capital asset pricing model to the Sharpe and TrosHos models and named the resulting models Sharpe CAPM and TrosHos CAPM respectively. After incoporating dynamic time series models to the Sharpe CAPM and TrosHos CAPM models we compared the resulting efficient frontiers and optimal portfolios.

There is evidence of serial autocorrelation and heteroskedasticity when using the single index model.The AR and Least Squares optimal portfolios include the same shares although the proportions invested in each share are different. This suggests that the serial autocorrelation has a small effect on our data set.

The GARCH and AR-GARCH optimal portfolio have the same shares although the proportion invested in each share are different. The optimal portfolio of the AR-GARCH and GARCH are different from the least squares models optimal portfolio which confirms that heteroskedasticity has an impact on the composition of shares in the optimal portfolio. The effect caused by serial autocorrelation does not appear to have a major impact in the composition of the optimal portfolios.

The TrosHos CAPM efficient frontier is constantly below the Sharpe CAPM efficient frontier for lower levels of risk. As the amount of risk increases the TrosHos CAPM efficient frontier shifts upwards and it eventually crosses the Sharpe CAPM efficient frontier. For high levels of risk the TrosHos CAPM efficient frontier is above the Sharpe CAPM efficient frontier.The explanation for this can be attributed to the differences in the covariance structure of the residuals used in the different models. The TrosHos CAPM model takes account of the all covariances (correlations) of the residuals while the Sharpe CAPM model does not take account of the covariance (correlation) of the residuals. The TrosHos CAPM efficient frontier contains more information compared to the Sharpe CAPM efficient frontier and by ignoring covariances (correlations) some risk is ignored by the Sharpe CAPM model.

The study extends the findings of Hossain et al. (2005), since in our study we incorporated the CAPM model on to the Sharpe and TrosHos single index models. In the same way as Hossain et al. (2005) we have displayed that the Sharpe CAPM model either underestimates or overestimates the risk of a portfolio where the covariance structure of the residuals is correlated. As a result of this fact the Sharpe model does not give a realistic account of the risk in the portfolio.

5 Summary of Conclusions

This section gives a summary of the findings from the different chapters:

Chapter 2: Markowitz Theory

The Markowitz portfolio selection model assumes that all investors are rational. For a given amount of risk investors will want to maximize return. For a given amount of return they will want to minimize risk. We constructed an optimal portfolio using the Markowitz formulation based on a risk-free rate of 8% p.a. using empirical data from the incorporate.

Chapter 3(a): The Capital Asset Pricing Model

The Capital Asset Pricing Model indicates a relationship between the price of a share and its risk. The CAPM model builds on the Markowitz model incorporating assumptions of the Markowitz model. Using empirical data from the incorporate, we were able to construct an optimal portfolio using the CAPM formulation. The optimal portfolio was very similar to the Markowitz optimal portfolio.

Chapter 3(b): The TrosHos CAPM and Sharpe CAPM

The TrosHos and Sharpe Single index models were extended to incorporate the Capital Asset Pricing model (CAPM). As indicated in chapter 3 the CAPM provides a valuable indication of the relationship between a share price and its level of risk. We named these models the TrosHos CAPM and Sharpe CAPM respectively. We made a comparison of the efficient frontiers and optimal portfolios of the TrosHos CAPM , Sharpe CAPM and Markowitz models. Despite the fact that the Markowitz model has different expected return inputs when compared to the TrosHos CAPM, they had very similar optimal portfolios. However, findings by Best and Grauer (1991) that portfolios are very sensitive to expected returns meant that comparison of these models does not provide a good guide to the risk-return relationship. The TrosHos CAPM and Sharpe CAPM models have the same expected return inputs. Their comparison is more valid than the comparison of the Sharpe CAPM and TrosHos CAPM models. The TrosHos efficient frontier was constantly below and to the left of the Sharpe CAPM efficient frontier for lower levels of risk. The TrosHos efficient frontier shifts upwards as we increase the portfolio risk levels. The TroHos CAPM efficient frontier eventually crosses the Sharpe CAPM efficient frontier and ends up above it as we increase the portfolio risk levels. The TrosHos CAPM models takes account of the correlation in the residuals whereas the Sharpe CAPM model does not.

Chapter 4: CAPM Dynamic Time Series Models

There was evidence of serial autocorrelation and heteroskedasticity in the index models. We used AR, GARCH and AR/GARCH models to model the autocorrelation and heteroskedasticity of the residuals. We compared the Least squares, AR , GARCH and AR/GARCH efficient frontiers and optimal portfolios of the TrosHos CAPM and Sharpe CAPM models. In Figures 6 to 9, we found the same behaviour of the efficient frontier of the TrosHos CAPM and Sharpe CAPM models. The TrosHos CAPM efficient frontier is below the Sharpe CAPM efficient frontier for lower levels of portfolio risk. As the portfolio risk is increased the TrosHos CAPM efficient frontier shifts upwards and eventually crosses the Sharpe CAPM efficient frontier. For large levels of portfolio risk the TrosHos CAPM efficient frontier is above Sharpe CAPM efficient frontier. The optimal portfolios of the dynamic time series models are different from that of the least squares model. The Sharpe CAPM does not provide a realistic account of the inherent risk of the portfolio since it does not account for the autocorrelation and heteroskedasticity found in the index models.

6 Future Research

The risk in the optimal portfolios of the TrosHos CAPM and Sharpe CAPM models can be broken down to market risk and unique risk. The market risk in the portfolios will be the same. However, the unique risk will be different due to the differences in the covariance structure of the residuals. In future the risk contribution to the optimal portfolio of the unique risk component could be investigated. The analysis can be categorized according to the different models: the least squares, AR, GARCH and AR/GARCH models under the TrosHos CAPM and Sharpe CAPM model formulation.

Mupambirei (2008) undertook a study on robust risk estimation under the Sharpe and TrosHos index models which could be extended to consider robust risk estimation under the Sharpe CAPM and TrosHos CAPM models.

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8 Appendices

8.1 Appendix 1: Chapter 4 Model Statistics

We document the adjusted R-Square, residual variance, Swartz information criteria, autocorrelation, partial autocorrelation and Q-statistics. These were used in model selection in Chapter 4.

Table 13: Adjusted R-Square of the Models

Share	**Least Squares**	**ARMA**	**GARCH**	ARMA&GARCH
Anglo	0.6866	0.6866	0.6874	0.6874
Impala Platinum	0.3502	0.3502	0.3502	0.3502
Pick and Pay	0.1486	0.2171	0.1486	0.2240
Remgro	0.3471	0.3601	0.3471	0.3646
ABSA	0.3196	0.3203	0.3196	0.3142
Richemont	0.3840	0.3840	0.3840	0.3840
Sasol	0.3995	0.3995	0.3995	0.3995
Tiger Brand	0.2523	0.2802	0.2523	0.2802
Afrox	0.2177	0.2325	0.2166	0.2345

Table 14: Residual Variance of the Models

Share	**Least Squares**	**ARMA**	**GARCH**	ARMA&GARCH
Anglo	0.0033	0.0033	0.0033	0.0033
Impala Platinum	0.0105	0.0105	0.0105	0.0105
Pick and Pay	0.0077	0.0071	0.0077	0.0072
Remgro	0.0037	0.0037	0.0037	0.0037
ABSA	0.0059	0.0060	0.0059	0.0061
Richemont	0.0035	0.0035	0.0035	0.0035
Sasol	0.0055	0.0055	0.0055	0.0055
Tiger Brand	0.0043	0.0041	0.0043	0.0042
Afrox	0.0052	0.0052	0.0052	0.0052

Table 15: Swartz Information Criterio of the Models

Share	**Least Squares**	**ARMA**	**GARCH**	ARMA&GARCH
Anglo	-2.8514	-2.8514	-2.9057	-2.9057
Impala Platinum	-1.6803	-1.6803	-1.6803	-1.6803
Pick and Pay	-1.9965	-2.0321	-1.9965	-2.0545
Remgro	-2.7172	-2.7166	-2.7172	-2.6920
ABSA	-2.2524	-2.2253	-2.2524	-2.2116
Richemont	-2.7900	-2.7900	-2.7900	-2.7900
Sasol	-2.3212	-2.3211	-2.3212	-2.3212
Tiger Brand	-2.5748	-2.6012	-2.5748	-2.6012
Afrox	-2.3895	-2.3559	-2.3921	-2.3616

Table 16: Autocorrelation and Partial Autocorrelation of Anglo American(JSE)

Lags	AC	PAC	Q-Stat
1	-0.028	-0.028	0.1913
2	-0.081	-0.082	1.8685
3	-0.005	-0.010	1.8745
4	0.099	0.093	4.3877
5	-0.071	-0.068	5.6879
6	-0.081	-0.071	7.3639
7	-0.008	-0.022	7.3807
8	0.038	0.016	7.7537
9	-0.025	-0.014	7.9199
10	-0.058	0.048	8.7952

Table 17: Autocorrelation and Partial Autocorrelation of Impala Platinum

Lags	AC	PAC	Q-Stat
1	-0.033	-0.033	0.2715
2	-0.037	-0.038	0.6120
3	0.059	0.057	1.5004
4	-0.017	-0.015	1.5784
5	0.012	0.015	1.6154
6	-0.051	-0.055	2.2872
7	-0.012	-0.012	2.3217
8	0.087	0.082	4.2996
9	0.002	0.013	4.3002
10	0.066	0.073	5.4378

Table 18: Autocorrelation and Partial Autocorrelation of Pick and Pay

Lags	AC	PAC	Q-Stat
1	-0.255	-0.255	16.394
2	0.116	0.054	19.772
3	-0.03	0.003	20.164
4	0.057	0.044	20.980
5	-0.175	-0.160	28.782
6	0.048	-0.041	29.372
7	-0.017	0.013	29.449
8	0.033	0.034	29.738
9	-0.137	-0.126	34.638
10	0.086	-0.008	36.561

Table 19: Autocorrelation and Partial Autocorrelation of Remgro

Lags	AC	PAC	Q-Stat
1	-0.156	-0.156	0.013
2	0.018	-0.007	0.044
3	-0.035	-0.034	0.088
4	0.012	0.001	0.160
5	-0.007	-0.005	0.253
6	-0.010	-0.013	0.359
7	0.005	0.002	0.470
8	0.002	0.003	0.579
9	0.025	0.026	0.660
10	0.034	0.043	0.718

Table 20: Autocorrelation and Partial Autocorrelation of ABSA

Lags	AC	PAC	Q-Stat
1	0.001	0.001	0.0005
2	0.010	0.010	0.0284
3	0.054	0.054	0.7635
4	-0.011	-0.011	0.7926
5	-0.087	-0.088	2.7239
6	-0.093	-0.096	4.9419
7	0.017	0.020	5.0137
8	0.003	0.016	5.0161
9	0.018	0.028	5.1051
10	-0.056	-0.070	5.9347

Table 21: Autocorrelation and Partial Autocorrelation of Richemont

Lags	AC	PAC	Q-Stat
1	-0.077	-0.077	1.4890
2	0.079	0.073	3.0603
3	0.051	0.063	3.7309
4	-0.025	-0.023	3.8886
5	0.057	0.046	4.7334
6	0.020	0.029	4.8355
7	-0.060	-0.063	5.7572
8	0.045	0.027	6.2894
9	-0.042	-0.028	6.7452
10	-0.021	-0.028	6.8628

Table 22: Atuocorrelation and Partial Autocorrelation of Sasol

Lags	AC	PAC	Q-Stat
1	0.079	0.079	1.5602
2	0.031	0.025	1.8083
3	-0.035	-0.039	2.1146
4	-0.015	-0.011	2.1751
5	-0.011	-0.007	2.2050
6	-0.011	-0.010	2.2340
7	-0.050	-0.049	2.8773
8	-0.021	-0.014	2.9926
9	-0.079	-0.075	4.6086
10	-0.074	-0.067	6.0444

Table 23: Autocorrelation and Partial Autocorrelation of Tiger Brand

Lags	AC	PAC	Q-Stat
1	-0.031	-0.031	0.2346
2	0.079	0.078	1.8089
3	-0.006	-0.001	1.8168
4	-0.085	-0.092	3.6592
5	0.007	0.003	3.6716
6	-0.214	-0.202	15.400
7	-0.045	-0.061	15.916
8	-0.103	-0.088	18.662
9	-0.028	-0.032	18.862
10	0.058	0.035	19.747

Table 24: Autocorrelation and Partial Autocorrelation of Afrox

Lags	AC	PAC	Q-Stat
1	-0.103	-0.103	2.6700
2	-0.065	-0.076	3.7391
3	0.103	0.089	6.4123
4	0.070	0.088	7.6713
5	-0.054	-0.025	8.4175
6	0.031	0.023	8.6640
7	-0.082	-0.099	10.390
8	-0.029	-0.045	10.605
9	-0.002	-0.019	10.606
10	0.09	0.109	13.082

8.2 Appendix 2: Eviews programing Code

This is the eviews program used to generate the expected returns, covariance of returns which we used as inputs in the Matlab program to compute efficient frontiers and optimal portfolios. This program was used for regression analysis. All the statistics in Appendix 1 were computed using this program.

```
workfile jse9apr 1 250
smpl 2 250
```

```
!n=249
!k=9
!p=8
!rf =800
scalar rfsa = !rf/10000
genr bay =ba/10000
genr rfm =log(1+bay)/12
scalar rf8 =log(1+.08)/12
'genr rfr =ba*rf12
genr r1=log(anglo) - log(anglo(-1))
genr q3 =log(jdgroup) - log(jdgroup(-1))
genr r2 = log(implat) - log(implat(-1))
genr r3 = log(picknp) - log(picknp(-1))
genr r4 = log(remgro) - log(remgro(-1))
genr r5 = log(absa) - log(absa(-1))
genr r6 = log(richem)- log(richem(-1))
genr r7 = log(sasol) - log(sasol(-1))
genr r8 = log(tigbran) - log(tigbran(-1))
genr q1 = log(tongat) - log(tongat(-1))
genr r9 = log(afrox) - log(afrox(-1))
genr x1 = log(anggol) - log(anggol(-1))
genr x2 = log(djtrans) - log(djtrans(-1))
genr x3 = log(djind) - log(djind(-1))
genr x4 = log(goldr) - log(goldr(-1))
genr x5 = log(angplats) - log(angplats(-1))
genr x6 = log(jseover) - log(jseover(-1))
genr x7 = log(palam) - log(palam(-1))
genr x8 = log(nedbank) - log(nedbank(-1))
genr x9 = log(ft100) - log(ft100(-1))
genr q2 = log(harmon) - log(harmon(-1))
```

```
genr q3 = log(implat) - log(implat(-1))
genr q4 = log(nampak) - log(nampak(-1))
genr q5 = log(ppc) - log(ppc(-1))
genr q6 = log(reunert) - log(reunert(-1))
genr q7 = log(reunert2) - log(reunert2(-1))
genr q8 = log(palam)- log(palam(-1))
' Nuw we start
genr incorporate = log(jseover) - log(jseover(-1))
scalar mjse = @mean(incorporate)
scalar vjse = @var(incorporate)
genr logang = log(anglo)
genr ang =r1
equation ang9.ls ang c x1 x2 x3 x4 x5 x6 x7 x8 x9
equation r4rem9.ls r4 c x1 x2 x3 x4 x5 x6 x7 x8 x9
equation r6rich9.ls r6 c x1 x2 x3 x4 x5 x6 x7 x8 x9
equation angarch.arch(1,1) ang c incorporate ar(2) ar(5)
equation angjse.ls ang c incorporate
equation angar.ls ang c incorporate ar(2) ar(5)
equation angsub.ls ang c x1 x2 x5 x6 x7 x8
equation angjse.ls ang c incorporate
vector(!k) mrls
mrls(1) =@mean(r1)
mrls(2) =@mean(r2)
mrls(3) =@mean(r3)
mrls(4) =@mean(r4)
mrls(5) =@mean(r5)
mrls(6) =@mean(r6)
mrls(7) =@mean(r7)
mrls(8) =@mean(r8)
mrls(9) =@mean(r9)
```

```
vector(!k) mqls
mqls(1) =@mean(q1)
mqls(2) =@mean(q2)
mqls(3) =@mean(q3)
mqls(4) =@mean(q4)
mqls(5) =@mean(q5)
mqls(6) =@mean(q6)
mqls(7) =@mean(q7)
mqls(8) =@mean(q8)
group rgrp r1 r2 r3 r4 r5 r6 r7 r8 r9
matrix(!k,!k) covmat
matrix(!k,!k) cormatm
covmat=@cov(rgrp)
cormatm =@cor(rgrp)
matrix rxmat = @convert(rgrp)
equation r1reg.ls r1 c incorporate
equation r2reg.ls r2 c incorporate
equation r3reg.ls r3 c incorporate
equation r4reg.ls r4 c incorporate
equation r5reg.ls r5 c jse
equation r6reg.ls r6 c jse
equation r7reg.ls r7 c jse
equation r8reg.ls r8 c jse
equation r9reg.ls r9 c jse
equation r1regar.ls r1 c jse ar(2) ar(4) ar(5)
equation r2regar.ls r2 c jse
equation r3regar.ls r3 c jse ar(1) ar(5)
equation r4regar.ls r4 c jse ar(1)
equation r5regar.ls r5 c jse ar(3) ar(5) ar(6)
equation r6regar.ls r6 c jse ar(1) ar(2) ar(3)
```

```
equation r7regar.ls r7 c jse ar(1)
equation r8regar.ls r8 c jse ar(2) ar(4) ar(6)
equation r9regar.ls r9 c jse ar(1) ar(2) ar(3) ar(4)
scalar y1bar = 0
vector(!k) s2arls
vector(!k) s2ls
s2arls(1)=r1regar.@se^2
s2arls(2)=r2regar.@se^2
s2arls(3)=r3regar.@se^2
s2arls(4)=r4regar.@se^2
s2arls(5)=r5regar.@se^2
s2arls(6)=r6regar.@se^2
s2arls(7)=r7regar.@se^2
s2arls(8)=r8regar.@se^2
s2arls(9)=r9regar.@se^2
s2ls(1)=r1reg.@se^2
s2ls(2)=r2reg.@se^2
s2ls(3)=r3reg.@se^2
s2ls(4)=r4reg.@se^2
s2ls(5)=r5reg.@se^2
s2ls(6)=r6reg.@se^2
s2ls(7)=r7reg.@se^2
s2ls(8)=r8reg.@se^2
s2ls(9)=r9reg.@se^2
' Matrix sigls =diag. LS. siglsar =DIAG AR matrix
matrix(!k,!k) siglsar
matrix(!k,!k) sigls
for !i=1 to !k
siglsar(!i,!i) = s2arls(!i)
sigls(!i,!i) = s2ls(!i)
```

```
next
vector(!k) bar
vector(!k) bls
bar(1) =r1regar.c(2)
bar(2) =r2regar.c(2)
bar(3) =r3regar.c(2)
bar(4) =r4regar.c(2)
bar(5)= r5regar.c(2)
bar(6) =r6regar.c(2)
bar(7) =r7regar.c(2)
bar(8) =r8regar.c(2)
bar(9) =r9regar.c(2)
bls(1) =r1reg.c(2)
bls(2) =r2reg.c(2)
bls(3) =r3reg.c(2)
bls(4) =r4reg.c(2)
bls(5)= r5reg.c(2)
bls(6) =r6reg.c(2)
bls(7) =r7reg.c(2)
bls(8) =r8reg.c(2)
bls(9) =r9reg.c(2)
r1reg.makeresid r1res
r2reg.makeresid r2res
r3reg.makeresid r3res
r4reg.makeresid r4res
r5reg.makeresid r5res
r6reg.makeresid r6res
r7reg.makeresid r7res
r8reg.makeresid r8res
r9reg.makeresid r9res
```

```
scalar m1res=@mean(r1res)
group rgrp r1res r2res r3res r4res r5res r6res r7res r8res r9res
matrix covres =@cov(rgrp)
scalar covdiags = 0
vector(!k) lsdi
for !i = 1 to !k
lsdi(!i)=covres(!i,!i)
covdiags = covdiags +covres(!i,!i)
next
scalar covoff = 0
for !j = 1 to !k-1
for !i = !j+1 to !k
covoff = covoff + covres(!i,!j)
next !i
next !j
scalar covpos = 0
scalar covneg = 0
for !j = 1 to !k-1
for !i = !j+1 to !k
if(covres(!i,!j)>0) then
covpos= covpos+ covres(!i,!j)
else
covneg =covneg +covres(!i,!j)
endif
next !i
next !j
scalar covoffc =covoff/covdiags*100
scalar covnegc =abs(covneg)/(abs(covneg)+covpos)*100
scalar covabs = covpos+abs(covneg)
scalar covabsc= covabs/covdiags*100
```

```
scalar covabsct=covabs/(covabs+covdiags)*100
scalar covtotal =covabs + covdiags
'vector(!k) pc1
'vector(!k) pc2
'freeze(tab1) rgrp.pcomp(cov,eigval = v1,eigvec=m1)
matrix covadj = @cov(rgrp)*!n/(!n-2)
matrix coradj = @cor(covadj)
matrix corres = @cor(rgrp)
matrix cormat = @cor(covres)
' Create Evar regressor residual variance matrix
matrix Evar=@convert(rgrp)
matrix Ermat = (@transpose(Evar)* Evar)/(!n)
matrix Ermata = @transpose(Evar)* (Evar)/(!n-2)
vector siglc = @getmaindiagonal(Ermat)
vector siglca = @getmaindiagonal(Ermata)
matrix siglcd = @makediagonal(siglc)
matrix siglcad = @makediagonal(siglca)
vector beta =bls
scalar vjseadj=vjse^(!n/(!n-1))
matrix sigvar = (beta*@transpose(beta))*vjse+ Ermat
matrix sigvara = (beta*@transpose(beta))*vjse+ Ermata
matrix sigdiag = (beta*@transpose(beta))*vjse + siglcd
matrix sigdiaga=(beta*@transpose(beta))*vjse+ siglcad
matrix sigdils = (beta*@transpose(beta))*vjse+ sigls
' Matrix sigls =diag. LS. siglsar =DIAG AR matrix
r1regar.makeresid r1arres
r2regar.makeresid r2arres
r3regar.makeresid r3arres
r4regar.makeresid r4arres
r5regar.makeresid r5arres
```

```
r6regar.makeresid r6arres
r7regar.makeresid r7arres
r8regar.makeresid r8arres
r9regar.makeresid r9arres
scalar m1ares=@mean(r1arres)
group argrp r1arres r2arres r3arres r4arres r5arres r6arres r7arres r8arres
r9arres
matrix covarm =@cov(argrp)
matrix covarma = covarm*((!n-6)/(!n-8))
' Create AEvar regressor residual AR variance matrix
matrix AEvar=@convert(argrp)
matrix AErmat =@transpose(AEvar)* AEvar/(!n-8)
matrix aecor = @cor(AEvar)
vector betar = bar
matrix sigarv = (betar*@transpose(betar))*vjse + AErmat
matrix sigdiar=(betar*@transpose(betar))*vjse+siglsar
matrix(!k,!k) AEcors
matrix AEcors= AErmat
for !i =1 to !k
matrix AEcors(!i,!i) = s2arls(!i)
next
matrix sigarc = (betar*@transpose(betar))*vjse + AEcors
' Matrix sigls =diag. LS. siglsar =DIAG AR matrix
equation r1jgar.arch(1,1) r1 c jse
equation r2jgar.arch(1,1) r2 c jse
equation r3jgar.arch(1,1) r3 c jse
equation r4jgar.arch(1,1) r4 c jse
equation r5jgar.arch(1,1) r5 c jse
equation r6jgar.arch(1,1) r6 c jse
equation r7jgar.arch(1,1) r7 c jse
```

```
equation r8jgar.arch(1,1) r8 c jse
equation r9jgar.arch(1,1) r9 c jse
r1jgar.makeresid r1jres
r2jgar.makeresid r2jres
r3jgar.makeresid r3jres
r4jgar.makeresid r4jres
r5jgar.makeresid r5jres
r6jgar.makeresid r6jres
r7jgar.makeresid r7jres
r8jgar.makeresid r8jres
r9jgar.makeresid r9jres
scalar mr1jres = @mean(r1jres)
scalar mr4jres = @mean(r4jres)
group jrgrp r1jres r2jres r3jres r4jres r5jres r6jres r7jres r8jres r9jres
" Create GJEvar residual GARCH only variance' 'matrix
matrix GJEvar=@convert(jrgrp)
matrix covjgar = @cov(jrgrp)
matrix covjmat = covjgar*(!n/(!n-5))
matrix GJErmat =@transpose(GJEvar)* (GJEvar)/(!n-5)
matrix GJEcor = @cor(GJEvar)
vector(!k) bejg
bejg(1) =r1jgar.c(2)
bejg(2) =r2jgar.c(2)
bejg(3) =r3jgar.c(2)
bejg(4) =r4jgar.c(2)
bejg(5) =r5jgar.c(2)
bejg(6) =r6jgar.c(2)
bejg(7) =r7jgar.c(2)
bejg(8) =r8jgar.c(2)
bejg(9) =r9jgar.c(2)
```

```
vector(!k) jgar
jgar(1) = r1jgar.@se^2
jgar(2) = r2jgar.@se^2
jgar(3) = r3jgar.@se^2
jgar(4) = r4jgar.@se^2
jgar(5) = r5jgar.@se^2
jgar(6) = r6jgar.@se^2
jgar(7) = r7jgar.@se^2
jgar(8) = r8jgar.@se^2
jgar(9) = r9jgar.@se^2
matrix(!k,!k) sigjar
for !i=1 to !k
sigjar(!i,!i) = jgar(!i)
next
matrix sigjgar = bejg*@transpose(bejg)*vjse + GJErmat
matrix sigjgara = bejg*@transpose(bejg)*vjse + covjmat
matrix sigdijar = bejg*@transpose(bejg)*vjse + sigjar
equation r1gar.arch(1,1) r1 c jse ar(2) ar(4) ar(5)
equation r2gar.arch(1,1) r2 c jse
equation r3gar.arch(1,1) r3 c jse ar(1) ar(5)
equation r4gar.arch(1,1) r4 c jse ar(1)
equation r5gar.arch(1,1) r5 c jse ar(3) ar(5) ar(6)
equation r6gar.arch(1,1) r6 c jse ar(1) ar(2) ar(3)
equation r7gar.arch(1,1) r7 c jse ar(1)
equation r8gar.arch(1,1) r8 c jse ar(2) ar(4) ar(6)
equation r9gar.arch(1,1) r9 c jse ar(1) ar(2) ar(3) ar(4)
equation q3gar.arch(1,1) q3 c jse
vector(!k) beg
beg(1) =r1gar.c(2)
beg(2) =r2gar.c(2)
```

```
beg(3) =r3gar.c(2)
beg(4) =r4gar.c(2)
beg(5) =r5gar.c(2)
beg(6) =r6gar.c(2)
beg(7) =r7gar.c(2)
beg(8) =r8gar.c(2)
beg(9) =r9gar.c(2)
r1gar.makeresid r1gres
r2gar.makeresid r2gres
r3gar.makeresid r3gres
r4gar.makeresid r4gres
r5gar.makeresid r5gres
r6gar.makeresid r6gres
r7gar.makeresid r7gres
r8gar.makeresid r8gres
r9gar.makeresid r9gres
vector(!k) sgar
sgar(1) = r1gar.@se^2
sgar(2) = r2gar.@se^2
sgar(3) = r3gar.@se^2
sgar(4) = r4gar.@se^2
sgar(5) = r5gar.@se^2
sgar(6) = r6gar.@se^2
sgar(7) = r7gar.@se^2
sgar(8) = r8gar.@se^2
sgar(9) = r9gar.@se^2
matrix(!k,!k) sigar
for !i = 1 to !k
sigar(!i,!i) = sgar(!i)
next
```

```
scalar sgres = r1gar.@ssr
group ggrp r1gres r2gres r3gres r4gres r5gres r6gres r7gres r8gres r9gres
matrix covgar = @cov(ggrp)
' Create GARvar residual GARCH AR variance matrix
matrix GARvar=@convert(ggrp)
matrix garcor2 = @cor(ggrp)
matrix garcor = @cor(GARvar)
matrix GARmat =@transpose(GARvar)*(GARvar)/(!n-11)
matrix covgara =covgar*((!n-6)/(!n-11))
matrix siggar = beg*@transpose(beg)*vjse + GARmat
matrix siggara = beg*@transpose(beg)*vjse + covgara
matrix sigdigar = beg*@transpose(beg)*vjse + sigar
matrix(!k,!k) GARcors
matrix GARcors =GARmat
for !i = 1 to !k
GARcors(!i,!i) = sgar(!I)
next
matrix siggarc = beg*@transpose(beg)*vjse + GARcors
vector(250) y1ar
vector(7) bag
bag(1) =r1regar.c(1)
bag(2) = r1regar.c(2)
bag(3) = r1regar.c(3)
bag(4) = r1regar.c(4)
bag(5) = r1regar.c(5)
for !i = 7 to 250
y1ar(!i) =c(1) +c(2)*jse(!i) + c(3)*jse(!i-3) +c(4)*jse(!i-5) +c(5)*jse(!i-6)
next
scalar mary1 =0
for !i = 7 to 250
```

```
mary1 = mary1 +y1ar(!i)
next
mary1 =mary1/(244)
scalar r1fbar = @mean(r1f)
scalar r1farm = @mean(r1far)
smpl 1 250
mtos(y1ar,y1ars)
vector(!k) mpred
mpred(1) = @mean(r1far)
mpred(2) = @mean(r2far)
mpred(3) =@mean(r3far)
mpred(4) =@mean(r4far)
mpred(5) =@mean(r5far)
mpred(6) =@mean(r6far)
mpred(7) =@mean(r7far)
mpred(8) =@mean(r8far)
mpred(9) =@mean(r9far)
vector(!k) mfgar
mfgar(1) = @mean(r1fgar)
mfgar(2) = @mean(r2fgar)
mfgar(3) = @mean(r3fgar)
mfgar(4) = @mean(r4fgar)
mfgar(5) = @mean(r5fgar)
mfgar(6) = @mean(r6fgar)
mfgar(7) = @mean(r7fgar)
mfgar(8) = @mean(r8fgar)
mfgar(9) = @mean(r9fgar)
'equation regq7.ls q7 c jse
equation r1cgar.arch(1,1) r1 c
equation r2cgar.arch(1,1) r2 c
```

```
equation r3cgar.arch(1,1) r3 c
equation r4cgar.arch(1,1) r4 c
equation r5cgar.arch(1,1) r5 c
equation r6cgar.arch(1,1) r6 c
equation r7cgar.arch(1,1) r7 c
equation r8cgar.arch(1,1) r8 c
equation r9cgar.arch(1,1) r9 c
vector(!k) mfcgar
scalar mfcgar1 = @mean(r1fcg)
mfcgar(1) = @mean(r1fcgar)
mfcgar(2) = @mean(r2fcgar)
mfcgar(3) = @mean(r3fcgar)
mfcgar(4) = @mean(r4fcgar)
mfcgar(5) = @mean(r5fcgar)
mfcgar(6) = @mean(r6fcgar)
mfcgar(7) = @mean(r7fcgar)
mfcgar(8) = @mean(r8fcgar)
mfcgar(9) = @mean(r9fcgar)
vector(!k) mfjgar
mfjgar(1) = @mean(r1fjgar)
mfjgar(2) = @mean(r2fjgar)
mfjgar(3) = @mean(r3fjgar)
mfjgar(4) = @mean(r4fjgar)
mfjgar(5) = @mean(r5fjgar)
mfjgar(6) = @mean(r6fjgar)
mfjgar(7) = @mean(r7fjgar)
mfjgar(8) = @mean(r8fjgar)
mfjgar(9) = @mean(r9fjgar)
vector(!k) mcong
mcong(1) = r1cgar.c(1)
```

```
mcong(2) = r2cgar.c(1)
mcong(3) = r3cgar.c(1)
mcong(4) = r4cgar.c(1)
mcong(5) = r5cgar.c(1)
mcong(6) = r6cgar.c(1)
mcong(7) = r7cgar.c(1)
mcong(8) = r8cgar.c(1)
mcong(9) = r9cgar.c(1)
r1cgar.makeresid r1cres
r2cgar.makeresid r2cres
r3cgar.makeresid r3cres
r4cgar.makeresid r4cres
r5cgar.makeresid r5cres
r6cgar.makeresid r6cres
r7cgar.makeresid r7cres
r8cgar.makeresid r8cres
r9cgar.makeresid r9cres
vector(!k) scgar
scgar(1) = r1cgar.@se^2
scgar(2) = r2cgar.@se^2
scgar(3) = r3cgar.@se^2
scgar(4) = r4cgar.@se^2
scgar(5) = r5cgar.@se^2
scgar(6) = r6cgar.@se^2
scgar(7) = r7cgar.@se^2
scgar(8) = r8cgar.@se^2
scgar(9) = r9cgar.@se^2
matrix(!k,!k) covcon
group gcres r1cres r2cres r3cres r4cres r5cres r6cres r7cres r8cres r9cres
matrix covcon =@cov(gcres)
```

```
' Create GARCvar residual GARCH CON variance matrix
matrix GARCvar=@convert(gcres)
matrix covcon2 = @cov(GARCvar)
matrix GARCmat =@transpose(GARCvar)*(GARCvar)/(!n-4)
scalar fact =!n/(!n-4)
matrix covconu = covcon*fact
equation regq1.ls q1 c jse
equation regq2.ls q2 c jse
equation regq3.ls q3 c jse
equation regq4.ls q4 c jse
equation regq5.ls q5 c jse
equation regq6.ls q6 c jse
equation regq7.ls q7 c jse
group rqgr r1 r2 r3 r4 r5 r6 r7 r8 r9 q1 q2 q3 q4 q5
vector(8) meanq
meanq(1) = @mean(q1)
meanq(2) = @mean(q2)
meanq(3) = @mean(q3)
meanq(4) = @mean(q4)
meanq(5) = @mean(q5)
meanq(6) = @mean(q6)
meanq(7) = @mean(q7)
meanq(8) = @mean(q8)
equation q1reg.ls q1 c jse
equation q2reg.ls q2 c jse
equation q3reg.ls q3 c jse
equation q4reg.ls q4 c jse
equation q5reg.ls q5 c jse
equation q6reg.ls q6 c jse
equation q7reg.ls q7 c jse
```

```
equation q8reg.ls q8 c jse
vector(8) seq
seq(1)=q1reg.@se^2
seq(2)=q2reg.@se^2
seq(3)=q3reg.@se^2
seq(4)=q4reg.@se^2
seq(5)=q5reg.@se^2
seq(6)=q6reg.@se^2
seq(7)=q7reg.@se^2
seq(8)=q8reg.@se^2
'genr richem = richem*.6162
scalar m6fcg =@mean(r6fcgar)
scalar m6 =@mean(r6)
scalar m6far = @mean(r6far)
scalar m6fgar =@mean(r6fgar)
scalar m6ls = @mean(r6fls)
scalar mr1cgar = @mean(r1cres)
scalar mr4cgar = @mean(r4cres)
scalar mr1ls = @mean(r1res)
scalar mr4ls = @mean(r4res)
scalar mr1ar = @mean(r1arres)
scalar mr4ar = @mean(r4arres)
scalar mr1gar = @mean(r1gres)
scalar mr4gar = @mean(r4gres)
scalar mr1jgar = @mean(r1jgres)
scalar mr4jgar = @mean(r4jgres)
scalar gsom =0
for !i =7 to !n
gsom =gsom +r1gres(!i)^2
next
```

```
scalar mg44 =gsom/244
scalar mg41 =gsom/241
genr r1gres2 =r1gres^2
scalar vargr1=@var(r1gres)
scalar varm =@mean(r1gres2)
scalar mr1j = @mean(r1jres)
equation q4regar.ls q4 c jse ar(1) ar(2) ar(3) ar(4)
equation q4regar.ls q4 c jse ar(1) ar(2) ar(3) ar(4)
equation q4gar.arch(1,1) q4 c jse ar(3) ar(4)
'SAM START
' Excess returns of securies
vector(!k) mrls
mrls(1) =@mean(r1)
mrls(2) =@mean(r2)
mrls(3) =@mean(r3)
mrls(4) =@mean(r4)
mrls(5) =@mean(r5)
mrls(6) =@mean(r6)
mrls(7) =@mean(r7)
mrls(8) =@mean(r8)
mrls(9) =@mean(r9)
scalar mjse = @mean(jse)
scalar vjse = @var(jse)
group rgrp r1 r2 r3 r4 r5 r6 r7 r8 r9
' covnat is Marko cov matrix
matrix(!k,!k) covmat
matrix(!k,!k) cormat
covmat=@cov(rgrp)
cormat =@cor(rgrp)
matrix rxmat = @convert(rgrp)
```

```
matrix covmatx =@cov(rxmat)
' We now use risk-free Rate as BA RATE(log)
genr r1c =r1-rfm
genr r2c =r2-rfm
genr r3c =r3-rfm
genr r4c =r4-rfm
genr r5c =r5-rfm
genr r6c =r6-rfm
genr r7c =r7-rfm
genr r8c =r8-rfm
genr r9c =r9-rfm
genr jsec =jse–rfm
vector(!k) mrf
mrf(1) =@mean(r1c)
mrf(2) =@mean(r2c)
mrf(3) =@mean(r3c)
mrf(4) =@mean(r4c)
mrf(5) =@mean(r5c)
mrf(6) =@mean(r6c)
mrf(7) =@mean(r7c)
mrf(8) =@mean(r8c)
mrf(9) =@mean(r9c)
' mrf is Marko risk-free adj. mean vector
group rgrpc r1c r2c r3c r4c r5c r6c r7c r8c r9c
' covnatc is Marko risk-free adj. cov matrix
matrix(!k,!k) covmatc
matrix(!k,!k) cormatc
covmatc=@cov(rgrpc)
cormatc =@cor(rgrpc)
matrix rxmatc = @convert(rgrpc)
```

```
matrix covmatxc =@cov(rxmatc)
matrix cormatxc =@cor(rxmatc)
equation r1regc.ls r1c c jsec
equation r2regc.ls r2c c jsec
equation r3regc.ls r3c c jsec
equation r4regc.ls r4c c jsec
equation r5regc.ls r5c c jsec
equation r6regc.ls r6c c jsec
equation r7regc.ls r7c c jsec
equation r8regc.ls r8c c jsec
equation r9regc.ls r9c c jsec
vector(!k) s2lsc
s2lsc(1)=r1regc.@se^2
s2lsc(2)=r2regc.@se^2
s2lsc(3)=r3regc.@se^2
s2lsc(4)=r4regc.@se^2
s2lsc(5)=r5regc.@se^2
s2lsc(6)=r6regc.@se^2
s2lsc(7)=r7regc.@se^2
s2lsc(8)=r8regc.@se^2
s2lsc(9)=r9regc.@se^2
' Matrix siglsc =diag. LS. sigarlsc =DIAG AR matrix
matrix(!k,!k) siglsc
for !i=1 to !k
siglsc(!i,!i) = s2lsc(!i)
next
vector(!k) blsc
blsc(1) =r1regc.c(2)
blsc(2) =r2regc.c(2)
blsc(3) =r3regc.c(2)
```

```
blsc(4) =r4regc.c(2)
blsc(5)= r5regc.c(2)
blsc(6) =r6regc.c(2)
blsc(7) =r7regc.c(2)
blsc(8) =r8regc.c(2)
blsc(9) =r9regc.c(2)
r1regc.makeresid r1resc
r2regc.makeresid r2resc
r3regc.makeresid r3resc
r4regc.makeresid r4resc
r5regc.makeresid r5resc
r6regc.makeresid r6resc
r7regc.makeresid r7resc
r8regc.makeresid r8resc
r9regc.makeresid r9resc
group rgrpc r1resc r2resc r3resc r4resc r5resc r6resc r7resc r8resc r9resc
matrix covresc =@cov(rgrpc)
matrix covadjc = @cov(rgrpc)*!n/(!n-2)
matrix coradjc = @cor(covadjc)
matrix corresc = @cor(rgrpc)
matrix cormat = @cor(covres)
' Create Evarc regressor residual variance matrix
matrix Evarc=@convert(rgrpc)
matrix Ermatac = @transpose(Evarc)* (Evarc)/(!n-2)
vector beta =blsc
scalar vjseadjc=vjse^(!n/(!n-1))
vector beta =blsc
'samavarac is the troskie hossain CAPM model'
matrix samvarac = (beta*@transpose(beta))*vjse+ Ermatac
```

'samdilsc is the Sharpe CAPM model and Markowitz is the original with mean mls and covmat'

```
matrix samdilsc = (beta*@transpose(beta))*vjse+ siglsc
equation r1regarc.ls r1c c jsec ar(2) ar(4) ar(5)
equation r2regarc.ls r2c c jsec
equation r3regarc.ls r3c c jsec ar(1) ar(5)
equation r4regarc.ls r4c c jsec ar(1)
equation r5regarc.ls r5c c jsec ar(3) ar(5) ar(6)
equation r6regarc.ls r6c c jsec ar(1) ar(2) ar(3)
equation r7regarc.ls r7c c jsec ar(1)
equation r8regarc.ls r8c c jsec ar(2) ar(4) ar(6)
equation r9regarc.ls r9c c jsec ar(1) ar(2) ar(3) ar(4)
vector(!k) s2arlsc
s2arlsc(1)=r1regarc.@se^2
s2arlsc(2)=r2regarc.@se^2
s2arlsc(3)=r3regarc.@se^2
s2arlsc(4)=r4regarc.@se^2
s2arlsc(5)=r5regarc.@se^2
s2arlsc(6)=r6regarc.@se^2
s2arlsc(7)=r7regarc.@se^2
s2arlsc(8)=r8regarc.@se^2
s2arlsc(9)=r9regarc.@se^2
vector(!k) barc
barc(1) =r1regarc.c(2)
barc(2) =r2regarc.c(2)
barc(3) =r3regarc.c(2)
barc(4) =r4regarc.c(2)
barc(5)= r5regarc.c(2)
barc(6) =r6regarc.c(2)
barc(7) =r7regarc.c(2)
```

```
barc(8) =r8regarc.c(2)
barc(9) =r9regarc.c(2)
r1regarc.makeresid r1arresc
r2regarc.makeresid r2arresc
r3regarc.makeresid r3arresc
r4regarc.makeresid r4arresc
r5regarc.makeresid r5arresc
r6regarc.makeresid r6arresc
r7regarc.makeresid r7arresc
r8regarc.makeresid r8arresc
r9regarc.makeresid r9arresc
group argrpc r1arresc r2arresc r3arresc r4arresc r5arresc r6arresc r7arresc
r8arresc r9arresc
matrix covarmc =@cov(argrpc)
matrix covarmac = covarmc*((!n-6)/(!n-8))
' Create AECvar regressor residual ARC variance matrix
matrix AECvar=@convert(argrpc)
matrix AECrmat =@transpose(AECvar)* AECvar/(!n-8)
matrix aecor = @cor(AECvar)
vector betar = barc
matrix sigarvc = (betar*@transpose(betar))*vjse + AECrmat
matrix(!k,!k) sigarlsc
for !i=1 to !k
sigarlsc(!i,!i) = s2arlsc(!i)
next
matrix sigdiarc=(betar*@transpose(betar))*vjse+sigarlsc
matrix(!k,!k) AECcors
matrix AECcors= AECrmat
for !i =1 to !k
matrix AECcors(!i,!i) = s2arlsc(!i)
```

```
next
matrix sigarcc = (betar*@transpose(betar))*vjse + AECcors
' Matrix sigls =diag. LS. sigarls =DIAG AR matrix
'We now do Samjgarc
equation r1jgarc.arch(1,1) r1 c jsec
equation r2jgarc.arch(1,1) r2 c jsec
equation r3jgarc.arch(1,1) r3 c jsec
equation r4jgarc.arch(1,1) r4 c jsec
equation r5jgarc.arch(1,1) r5 c jsec
equation r6jgarc.arch(1,1) r6 c jsec
equation r7jgarc.arch(1,1) r7 c jsec
equation r8jgarc.arch(1,1) r8 c jsec
equation r9jgarc.arch(1,1) r9 c jsec
r1jgarc.makeresid r1jresc
r2jgarc.makeresid r2jresc
r3jgarc.makeresid r3jresc
r4jgarc.makeresid r4jresc
r5jgarc.makeresid r5jresc
r6jgarc.makeresid r6jresc
r7jgarc.makeresid r7jresc
r8jgarc.makeresid r8jresc
r9jgarc.makeresid r9jresc
scalar mr1jresc = @mean(r1jresc)
scalar mr4jresc = @mean(r4jresc)
group jrgrpc r1jresc r2jresc r3jresc r4jresc r5jresc r6jresc r7jresc r8jresc
r9jresc
" Create GJCEvar residual GARCH JSEC only variance' 'matrix
matrix GJCEvar=@convert(jrgrpc)
matrix covjgarc = @cov(jrgrpc)
matrix covjmatc = covjgarc*(!n/(!n-5))
```

```
matrix GJCErmat =@transpose(GJCEvar)* (GJCEvar)/(!n-5)
matrix GJCEcor = @cor(jrgrpc)
vector(!k) bejgc
bejgc(1) =r1jgarc.c(2)
bejgc(2) =r2jgarc.c(2)
bejgc(3) =r3jgarc.c(2)
bejgc(4) =r4jgarc.c(2)
bejgc(5) =r5jgarc.c(2)
bejgc(6) =r6jgarc.c(2)
bejgc(7) =r7jgarc.c(2)
bejgc(8) =r8jgarc.c(2)
bejgc(9) =r9jgarc.c(2)
vector(!k) jgarc
jgarc(1) = r1jgarc.@se^2
jgarc(2) = r2jgarc.@se^2
jgarc(3) = r3jgarc.@se^2
jgarc(4) = r4jgarc.@se^2
jgarc(5) = r5jgarc.@se^2
jgarc(6) = r6jgarc.@se^2
jgarc(7) = r7jgarc.@se^2
jgarc(8) = r8jgarc.@se^2
jgarc(9) = r9jgarc.@se^2
matrix(!k,!k) sigjarc
for !i=1 to !k
sigjarc(!i,!i) = jgarc(!i)
next
matrix sigjgarc = bejgc*@transpose(bejgc)*vjse + GJCErmat
matrix sigjgarac = bejgc*@transpose(bejgc)*vjse + covjmatc
matrix sigdijarc = bejgc*@transpose(bejgc)*vjse + sigjarc
equation r1garc.arch(1,1) r1c c jsec ar(2) ar(4) ar(5)
```

```
equation r2garc.arch(1,1) r2c c jsec
equation r3garc.arch(1,1) r3c c jsec ar(1) ar(5)
equation r4garc.arch(1,1) r4c c jsec ar(1)
equation r5garc.arch(1,1) r5c c jsec ar(3) ar(5) ar(6)
equation r6garc.arch(1,1) r6c c jsec ar(1) ar(2) ar(3)
equation r7garc.arch(1,1) r7c c jsec ar(1)
equation r8garc.arch(1,1) r8c c jsec ar(2) ar(4) ar(6)
equation r9garc.arch(1,1) r9c c jsec ar(1) ar(2) ar(3) ar(4)
equation q3gar.arch(1,1) q3 c jse
vector(!k) begc
begc(1) =r1garc.c(2)
begc(2) =r2garc.c(2)
begc(3) =r3garc.c(2)
begc(4) =r4garc.c(2)
begc(5) =r5garc.c(2)
begc(6) =r6garc.c(2)
begc(7) =r7garc.c(2)
begc(8) =r8garc.c(2)
begc(9) =r9garc.c(2)
r1garc.makeresid r1gresc
r2garc.makeresid r2gresc
r3garc.makeresid r3gresc
r4garc.makeresid r4gresc
r5garc.makeresid r5gresc
r6garc.makeresid r6gresc
r7garc.makeresid r7gresc
r8garc.makeresid r8gresc
r9garc.makeresid r9gresc
vector(!k) sgarc
sgarc(1) = r1garc.@se^2
```

```
sgarc(2) = r2garc.@se^2
sgarc(3) = r3garc.@se^2
sgarc(4) = r4garc.@se^2
sgarc(5) = r5garc.@se^2
sgarc(6) = r6garc.@se^2
sgarc(7) = r7garc.@se^2
sgarc(8) = r8garc.@se^2
sgarc(9) = r9garc.@se^2
matrix(!k,!k) sigarc
for !i = 1 to !k
sigarc(!i,!i) = sgarc(!i)
next
scalar sgresc = r1garc.@ssr
group ggrpc r1gresc r2gresc r3gresc r4gresc r5gresc r6gresc r7gresc r8gresc
r9gresc
matrix covgarc = @cov(ggrpc)
' Create GARCvar res. jsec GARCH AR resvar. matrix
matrix GARCvar=@convert(ggrpc)
matrix garcor2c = @cor(ggrpc)
matrix garcorc = @cor(GARCvar)
matrix GARCmat =@transpose(GARCvar)*(GARCvar)/(!n-11)
matrix covgarac =covgarc*((!n-6)/(!n-11))
matrix siggarc = begc*@transpose(begc)*vjse + GARCmat
matrix siggarac = begc*@transpose(begc)*vjse + covgarac
matrix sigdigarc=begc*@transpose(begc)*vjse + sigarc
matrix(!k,!k) GARCcors
matrix GARcors =GARCmat
for !i = 1 to !k
GARccors(!i,!i) = sgarc(!I)
next
```

```
matrix siggarcc = begc*@transpose(begc)*vjse + GARCcors
vector(250) y1ar
vector(7) bag
bag(1) =r1regar.c(1)
bag(2) = r1regar.c(2)
bag(3) = r1regar.c(3)
bag(4) = r1regar.c(4)
bag(5) = r1regar.c(5)
for !i = 7 to 250
y1ar(!i) =c(1) +c(2)*jse(!i) + c(3)*jse(!i-3) +c(4)*jse(!i-5) +c(5)*jse(!i-6)
next
scalar mary1 =0
for !i = 7 to 250
mary1 = mary1 +y1ar(!i)
next
mary1 =mary1/(244)
scalar r1fbar = @mean(r1f)
scalar r1farm = @mean(r1far)
smpl 1 250
mtos(y1ar,y1ars)
vector(!k) mpred
mpred(1) = @mean(r1far)
mpred(2) = @mean(r2far)
mpred(3) =@mean(r3far)
mpred(4) =@mean(r4far)
mpred(5) =@mean(r5far)
mpred(6) =@mean(r6far)
mpred(7) =@mean(r7far)
mpred(8) =@mean(r8far)
mpred(9) =@mean(r9far)
```

```
vector(!k) mfgar
mfgar(1) = @mean(r1fgar)
mfgar(2) = @mean(r2fgar)
mfgar(3) = @mean(r3fgar)
mfgar(4) = @mean(r4fgar)
mfgar(5) = @mean(r5fgar)
mfgar(6) = @mean(r6fgar)
mfgar(7) = @mean(r7fgar)
mfgar(8) = @mean(r8fgar)
mfgar(9) = @mean(r9fgar)
'equation regq7.ls q7 c jse
equation r1cgar.arch(1,1) r1 c
equation r2cgar.arch(1,1) r2 c
equation r3cgar.arch(1,1) r3 c
equation r4cgar.arch(1,1) r4 c
equation r5cgar.arch(1,1) r5 c
equation r6cgar.arch(1,1) r6 c
equation r7cgar.arch(1,1) r7 c
equation r8cgar.arch(1,1) r8 c
equation r9cgar.arch(1,1) r9 c
vector(!k) mfcgar
scalar mfcgar1 = @mean(r1fcg)
mfcgar(1) = @mean(r1fcgar)
mfcgar(2) = @mean(r2fcgar)
mfcgar(3) = @mean(r3fcgar)
mfcgar(4) = @mean(r4fcgar)
mfcgar(5) = @mean(r5fcgar)
mfcgar(6) = @mean(r6fcgar)
mfcgar(7) = @mean(r7fcgar)
mfcgar(8) = @mean(r8fcgar)
```

```
mfcgar(9) = @mean(r9fcgar)
vector(!k) mfjgar
vector(!k) mgresc
mgresc(1) =@mean(r1gresc)
mgresc(2) =@mean(r2gresc)
mgresc(3) =@mean(r3gresc)
mgresc(4) =@mean(r4gresc)
mgresc(5) =@mean(r5gresc)
mgresc(6) =@mean(r6gresc)
mgresc(7) =@mean(r7gresc)
mgresc(8) =@mean(r8gresc)
mgresc(9) =@mean(r9gresc)
vector(!k) mgres
mgres(1) =@mean(r1gres)
mgres(2) =@mean(r2gres)
mgres(3) =@mean(r3gres)
mgres(4) =@mean(r4gres)
mgres(5) =@mean(r5gres)
mgres(6) =@mean(r6gres)
mgres(7) =@mean(r7gres)
mgres(8) =@mean(r8gres)
mgres(9) =@mean(r9gres)
vector(!k) mjres
mjres(1) =@mean(r1jres)
mjres(2) =@mean(r2jres)
mjres(3) =@mean(r3jres)
mjres(4) =@mean(r4jres)
mjres(5) =@mean(r5jres)
mjres(6) =@mean(r6jres)
mjres(7) =@mean(r7jres)
```

mjres(8) =@mean(r8jres)
mjres(9) =@mean(r9jres)

8.3 Appendix 3: Matlab Program

This is the Matlab program used to draw the different efficient frontiers and optimal portfolios. The inputs into this program were generate in the E-views program in Appendix 2.This is program produces the TrosHos GARCH efficient frontier and optimal portfolio.

```
ExpReturn=[
 0.01040  0.01530  0.01160  0.01670  0.01410  0.01190  0.01400  0.01080  0.00910

];
ExpCovariance = [

 0.01066   0.00848   0.00144  0.00312  0.00259  0.00386  0.00602  0.00246  0.00266;
 0.00847   0.01778  -0.00055  0.00242  0.00167  0.00421  0.00476  0.00130  0.00198;
 0.00143  -0.00055   0.00881  0.00218  0.00366  0.00121  0.00091  0.00294  0.00186;
 0.00312   0.00242   0.00218  0.00554  0.00327  0.00272  0.00195  0.00272  0.00233;
 0.00259   0.00167   0.00366  0.00327  0.00839  0.00228  0.00164  0.00312  0.00248;
 0.00386   0.00421   0.00121  0.00272  0.00228  0.00575  0.00270  0.00159  0.00184;
 0.00601   0.00476   0.00091  0.00195  0.00164  0.00270  0.00919  0.00133  0.00158;
 0.00246   0.00130   0.00294  0.00272  0.00312  0.00159  0.00133  0.00583  0.00199;
 0.00266   0.00198   0.00186  0.00233  0.00248  0.00184  0.00158  0.00199  0.00653;
]

NumPorts = 20;
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,ExpCovariance,
NumPorts);
RisklessRate = 0.0024632
BorrowRate = NaN
RiskAversion = 3;
portalloc (PortRisk, PortReturn, PortWts, RisklessRate,BorrowRate, RiskAver-
sion);
```

[RiskyRisk, RiskyReturn, RiskyWts,RiskyFraction, OverallRisk,OverallReturn] = portalloc (PortRisk, PortReturn, PortWts,RisklessRate, BorrowRate, RiskAversion)